Computation of the Power Line Electric and Magnetic Fields

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***Abstract* — This paper presents a novel numerical algorithm for computing of the power line electric and magnetic fields. Phase conductor currents are separated into their longitudinal and transversal components. Computation of the magnetic flux density is based on the Biot-Savart law. Longitudinal component of the electric field intensity is computed from vector magnetic potential. Two other components of the electric field intensity are computed from the transversal currents, which are obtained by the average potential method.**

***Key words* — Electric field, magnetic field, overhead power line, buried cable line, Biot-Savart law, average potential method**

I. INTRODUCTION

THE question of electromagnetic pollution is becoming a matter of serious scientific and public health policy concern with the development of general consciousness about the possible adverse human health effects of electromagnetic fields. The most controversial one is the supposed elevated risk of cancer.

The ICINIRP guidelines [1] recommended reference levels for 50 Hz occupational exposure are 500 μT for the magnetic flux density and 10 kV/m for the electric field intensity. For general public exposure to 50 Hz electromagnetic field, the reference levels are 100 μT for the magnetic flux density and 5 kV/m for the electric field intensity.

In numerical models for computation of the power line electromagnetic field, the power frequency electric and magnetic field computation is based on the quasi-static approximation [2], [3].

II. SCALAR ELECTRIC AND VECTOR MAGNETIC POTENTIALS

In mathematical model developed for the computation of the electric and magnetic field of power line straight section, phase conductors of the power line satisfy a thin-wire approximation and are treated as line sources positioned parallel to the earth surface [4] - [6]. Number of line sources equals the number of phase conductors, which are positioned in the air or earth at some distance from the surface. Line sources are oriented along the *x*-axis of the Cartesian coordinate system. Origin of the selected coordinate system is on the earth surface and in the middle of power line section. Cross section of a typical three-phase high voltage power line with a single shield wire and a buried cable line, arranged in the *y*-*z* plane of the selected coordinate system is presented in Fig. 1.

Computation of the electric and magnetic field is carried out in the *y*-*z* plane positioned in the middle of the power line section.

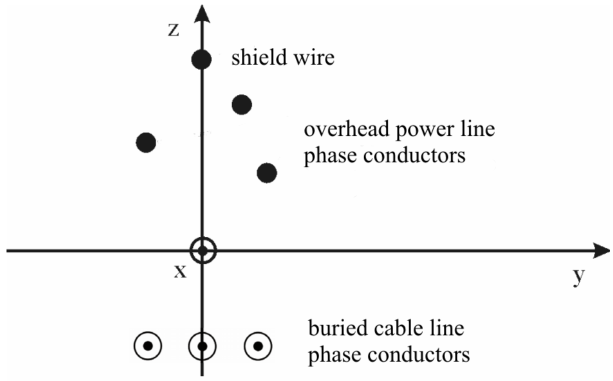

Fig. 1. Position of an overhead and buried cable power line in the coordinate system.

Current which flows through each of the phase conductors can be, according to [3], [4], [6], separated into two individual parts: longitudinal phase conductor current and transversal phase conductor current. According to the adopted thin-wire approximation, longitudinal current is flowing along the phase conductor axis. This current is approximated by a constant, whose value is equal to its average value. Transversal (leakage) current leaks uniformly from the surface of the phase conductor into the surrounding medium (air or earth).

Starting point for the development of the mathematical model for the computation of electromagnetic field of power lines can be seen in solutions of the Helmholtz differential equation for the scalar electric and vector magnetic potentials [3], [5]. Solutions of the Helmholtz differential equation for the scalar electric and vector magnetic potentials in the homogeneous, linear, isotropic and unbounded medium, with neglected potential retardation effects reduce to the solutions of Poisson differential equations [4], [6]. Retardation effects can be neglected in this case, without loss of accuracy, due to the

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fact that frequency of interest equals the power frequency (50 – 60 Hz). By introducing the phase conductor longitudinal and transversal currents, in a case of arbitrary number of phase conductors (n), solutions of the Poisson differential equations for the scalar electric and vector magnetic potentials can be expressed by [3], [4], [6]:

$$\overline{\varphi} = \frac{1}{4\cdot\pi\cdot\overline{\kappa}}\cdot\sum_{i=1}^{n}\frac{\overline{I}_i^t}{\ell}\cdot\int_{\Gamma_i}\frac{1}{R}\cdot d\ell_i \tag{1}$$

$$\dot{\vec{A}} = \vec{i}\cdot\frac{\mu_0}{4\cdot\pi}\cdot\sum_{i=1}^{n}\overline{I}_i^\ell\cdot\int_{\Gamma_i}\frac{1}{R}\cdot d\ell_i \tag{2}$$

where $\overline{\kappa}$ is the complex electrical conductivity of the medium, $\overline{I}_{\mathrm{i}}^\ell$ is the longitudinal current of the i^{th} phase conductor, $\overline{I}_{\mathrm{i}}^t$ is the transversal current of the i^{th} phase conductor, ℓ is the length of the power line section and R is the distance between the field and source points. Integration path Γ_i in the equations (1) and (2) is positioned along the i^{th} phase conductor axis.

The longitudinal currents are the input data in all cases. The transversal phase conductor currents are non-existent in buried cable lines, while transversal phase conductor currents in overhead power lines are computed by the well-known average potential method [7], which gives the following system of linear algebraic equations:

$$\sum_{i=1}^{n}\overline{Z}_{ji}\cdot\overline{I}_i^t = \overline{\Phi}_j \quad ; \quad j=1,2,\ldots,n \tag{3}$$

where $\overline{\Phi}_j$ is the average potential of the j^{th} phase conductor. The shield wire of the overhead power line is just a special case of the phase conductor with $\overline{I}_{\mathrm{i}}^\ell = 0$ and $\overline{\Phi}_i = 0$.

Impedances $\overline{Z}_{ji}$ from the equation (3) can be computed using the following expression:

$$\overline{Z}_{\mathrm{ji}} = \overline{Z}(d_{ji}) + \overline{F}\cdot\overline{Z}(D_{\mathrm{ji}}) \tag{4}$$

where $\overline{F}$ is a reflection coefficient, which can be expressed as follows [3], [4], [8]:

$$\overline{F} = \frac{\overline{\kappa}_1 - \overline{\kappa}_2}{\overline{\kappa}_1 + \overline{\kappa}_2} = \frac{j\cdot 2\cdot\pi\cdot f\cdot\varepsilon_0 - (\sigma + j\cdot 2\cdot\pi\cdot f\cdot\varepsilon_0\cdot\varepsilon_r)}{j\cdot 2\cdot\pi\cdot f\cdot\varepsilon_0 + (\sigma + j\cdot 2\cdot\pi\cdot f\cdot\varepsilon_0\cdot\varepsilon_r)} \tag{5}$$

with $\overline{\kappa}_1$ and $\overline{\kappa}_2$ representing the complex electrical conductivities of the air and earth, respectively, σ is the electrical conductivity of the earth, f is the frequency, $\varepsilon_0 = 8.854\cdot 10^{-12}$ F/m is the vacuum permittivity and ε_r is the relative permittivity of the earth.

Impedances $\overline{Z}(d_{ji})$ and $\overline{Z}(D_{ji})$ from the expression (4) can be computed using the following equation [3]:

$$\overline{Z}(\mathrm{v}) = \frac{1}{2\cdot\pi\,\overline{\kappa}_1\cdot\ell^2}\left[\ell\cdot\ell\mathrm{n}\frac{\sqrt{\ell^2+v^2}+\ell}{v} - \sqrt{\ell^2+v^2} + v\right] \tag{6}$$

by introducing the following values instead of v:

$$d_{ii} = r_{0i}\,;\quad D_{ii} = 2\cdot z_i \tag{7}$$

$$d_{ji} = \sqrt{(y_i - y_j)^2 + (z_i - z_j)^2} \qquad i \neq j \tag{8}$$

$$D_{ji} = \sqrt{(y_i - y_j)^2 + (z_i + z_j)^2} \qquad i \neq j \tag{9}$$

where r_{0i} is a radius of the i^{th} conductor, while y_i and z_i are coordinates of the i^{th} conductor middle point.

Numerical algorithm for the computation of electromagnetic field of power lines is hereafter subdivided into two separate parts:

- numerical algorithm for the computation of magnetic flux density,
- numerical algorithm for the computation of electric field intensity.

III. Computation of the Power Line Magnetic Flux Density

Numerical algorithm for the computation of the magnetic flux density of the overhead power line and buried cable line are mathematically identical and are based on the application of the Biot-Savart law [2]. Situation of the arbitrary i^{th} phase conductor of the high voltage power line positioned in the x direction of the selected coordinate system is depicted in the Fig. 2. This phase conductor is carrying a longitudinal current $\overline{I}_{\mathrm{i}}^\ell$, which causes the magnetic field at the observation point P(0, y, z) in the y-z plane, also depicted in the Fig. 2.

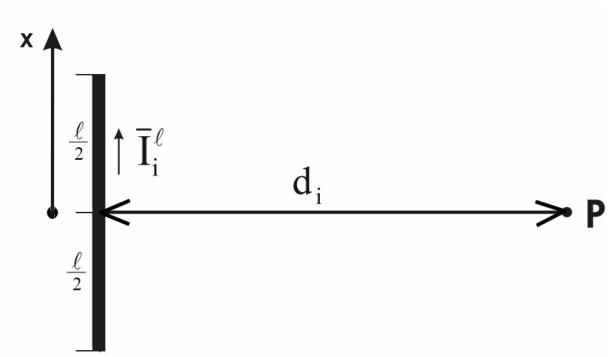

Fig. 2. Computing of the magnetic field caused by a single phase conductor of the power line.

For power line, components of the magnetic flux density are computed for the observation point P(0, y, z) in the y-z plane using the following expressions [3]:

$$\overline{B}_x = 0 \tag{10}$$

$$\overline{B}_{\mathrm{y}} = \mu_0\cdot\sum_{i=1}^{n}\frac{(z_i - z)\cdot\ell}{4\cdot\pi\cdot d_i^2\cdot\sqrt{d_i^2 + \frac{\ell^2}{4}}}\cdot\overline{I}_{\mathrm{i}}^\ell \tag{11}$$

$$\overline{B}_{\mathrm{z}} = \mu_0\cdot\sum_{i=1}^{n}\frac{(y - y_i)\cdot\ell}{4\cdot\pi\cdot d_i^2\cdot\sqrt{d_i^2 + \frac{\ell^2}{4}}}\cdot\overline{I}_{\mathrm{i}}^\ell \tag{12}$$

where:

$$d_i = \sqrt{(y - y_i)^2 + (z - z_i)^2} \tag{13}$$

The constant $\mu_0 = 4\cdot\pi\cdot 10^{-7}$ H/m is the medium permeability, which is equal to vacuum permeability. Finally, the effective total value of the magnetic flux density at the observed point P can be written as:

$$B = \sqrt{\left|\overline{B}_{\mathrm{y}}\right|^2 + \left|\overline{B}_{\mathrm{z}}\right|^2} \tag{14}$$

IV. Computation of the Overhead Power Line Electric Field Intensity

Computation of the overhead power line electric field intensity can be sought through the well-known equation which combines scalar electric potential and vector magnetic potential [2]-[4]:

$$\dot{\bar{E}} = -\nabla\bar{\varphi} - j\cdot\omega\cdot\dot{\bar{A}} = \{\bar{E}_x, \bar{E}_y, \bar{E}_z\} \tag{15}$$

Unlike the computation of the magnetic field, which was equivalent for both overhead and buried cable power line, this is not the case for the computation of electric field intensity.

In the here presented case of the overhead power line, vector magnetic potential has only the x-component, while potential distribution is symmetric in respect to the y-z plane. Components of the electric field intensity for the overhead power line can be computed from the following expressions [3]:

$$\bar{E}_x = \bar{E}_x\big|_{x=0} = -j\cdot\omega\cdot\bar{A}_x = -j\cdot\omega\cdot\bar{A}\Big|_{x=0} \tag{16}$$

$$\bar{E}_y = \bar{E}_y\big|_{x=0} = -\frac{\partial\bar{\varphi}(x,y,z)}{\partial y}\Big|_{x=0} = -\frac{\partial\bar{\varphi}(0,y,z)}{\partial y} \tag{17}$$

$$\bar{E}_z = \bar{E}_z\big|_{x=0} = -\frac{\partial\bar{\varphi}(x,y,z)}{\partial z}\Big|_{x=0} = -\frac{\partial\bar{\varphi}(0,y,z)}{\partial z} \tag{18}$$

where partial derivatives of the scalar electric potential features prominently in the development of y- and z-components. It can be seen from the expressions (17) and (18) that potential distribution is only needed in the y-z plane (x = 0), due to the fact that potential distribution is symmetric in respect to this plane. Magnetic vector potential at the y-z plane, which is a consequence of n phase conductor longitudinal currents, can be expressed as follows [3], [4]:

$$\bar{A}\Big|_{x=0} = \frac{\mu_0}{2\cdot\pi}\sum_{i=1}^{n}\bar{I}_i^{\ell}\cdot\ell n\frac{\sqrt{d_i^2+\frac{\ell^2}{4}}+\frac{\ell}{2}}{d_i} \tag{19}$$

Potential distribution in the y-z plane due to the transversal currents of all phase conductors can be expressed according to the following expression [3]:

$$\bar{\varphi}(0,y,z) = \frac{1}{4\cdot\pi\cdot\bar{\kappa}_1\cdot\ell}\cdot\sum_{i=1}^{n}\left[G(0,d_i)+\bar{F}\cdot G(0,D_i)\right]\cdot\bar{I}_i^t \tag{20}$$

where:

$$G(0,v) = 2\cdot\sinh^{-1}\left(\frac{\ell}{2\cdot v}\right) \quad ; \; v\in\{d_i, D_i\} \tag{21}$$

The newly introduced variable D_i is the shortest distance between the observation point and the line passing through the image of the i^{th} phase conductor axis; it can be computed as follows:

$$D_i = \sqrt{(y-y_i)^2+(z+z_i)^2} \tag{22}$$

Finally, computation of electric field intensity for the overhead power line can be accomplished with the following expressions [3]:

$$\bar{E}_x = -\frac{j\cdot\omega\cdot\mu_0}{2\cdot\pi}\sum_{i=1}^{n}\bar{I}_i^{\ell}\cdot\ell n\frac{\sqrt{d_i^2+\frac{\ell^2}{4}}+\frac{\ell}{2}}{d_i} \tag{23}$$

$$\bar{E}_y = -\frac{1}{4\cdot\pi\cdot\bar{\kappa}_1\cdot\ell}\cdot\sum_{i=1}^{n}\left[\frac{\partial G(0,d_i)}{\partial d_i}\cdot\frac{\partial d_i}{\partial y}+\bar{F}\cdot\frac{\partial G(0,D_i)}{\partial D_i}\cdot\frac{\partial D_i}{\partial y}\right]\cdot\bar{I}_i^t \tag{24}$$

$$\bar{E}_z = -\frac{1}{4\cdot\pi\cdot\bar{\kappa}_1\cdot\ell}\cdot\sum_{i=1}^{n}\left[\frac{\partial G(0,d_i)}{\partial d_i}\cdot\frac{\partial d_i}{\partial z}+\bar{F}\cdot\frac{\partial G(0,D_i)}{\partial D_i}\cdot\frac{\partial D_i}{\partial z}\right]\cdot\bar{I}_i^t \tag{25}$$

where:

$$\frac{\partial G(0,v)}{\partial v} = -\,2\cdot\frac{\ell}{v}\cdot\frac{1}{\sqrt{4\cdot v^2+\ell^2}} \quad ; \; v\in\{d_i, D_i\} \tag{26}$$

Total effective value of the electric field intensity of the overhead line at the observed point P can be written as:

$$E = \sqrt{\left|\bar{E}_x\right|^2+\left|\bar{E}_y\right|^2+\left|\bar{E}_z\right|^2} \tag{27}$$

V. Computation of the Buried Cable Line Electric Field Intensity

The computation of the electric field intensity of buried cable lines is a special case of the above presented algorithm for the computation of the electric field intensity of overhead power lines. Since there is no transversal component of the current outside of the buried cable line, the scalar potential in the air and earth is non-existent so the expression (15) for the computation of cable line electric field intensity becomes:

$$\dot{\bar{E}} = -j\cdot\omega\cdot\dot{\bar{A}} = \{\bar{E}_x, 0, 0\} \tag{28}$$

The x-component of the electric field intensity, described by (23), is mathematically identical to the one presented for the overhead power line case.

VI. Numerical Examples

Two numerical examples will be presented which demonstrate computation of the electromagnetic field for a typical overhead power line and for a buried cable line.

A. Overhead power line

This numerical example features a typical 110 kV overhead power line. The input data and the geometry of the representative power line tower are given in Table 1.

Table 1: Input data for overhead power line.

	i	y(m)	z(m)	$\bar{\varphi}$ (kV)	$\bar{I}^{\ell}$ (A)
Shield wire	1	0	27.9	$0\angle 0°$	$0\angle 0°$
L1	2	2.5	24.8	$63.5\angle 0°$	$630\angle -30°$
L2	3	-3.0	22.4	$63.5\angle -120°$	$630\angle -150°$
L3	4	3.5	20.0	$63.5\angle 120°$	$630\angle 90°$

The data in Table 1, together with the power frequency f = 50 Hz, length of the overhead power line ℓ = 1 km, radius of the shield wire r_{01} = 4.72 mm, radii of the phase conductors $r_{02} = r_{03} = r_{04}$ = 10.95 mm, relative permittivity of earth ε_r = 10 and the electrical conductivity of earth σ = 0.01 S/m represent the input data for the numerical example. Computation of the electromagnetic field is carried out at observation points along y-directed profile, positioned at 1 m above the earth surface (z = 1 m).

Fig. 3 and Fig. 4 respectively present computed effective values of the components and the total effective values of magnetic flux density and electric field intensity, along the observation profile.

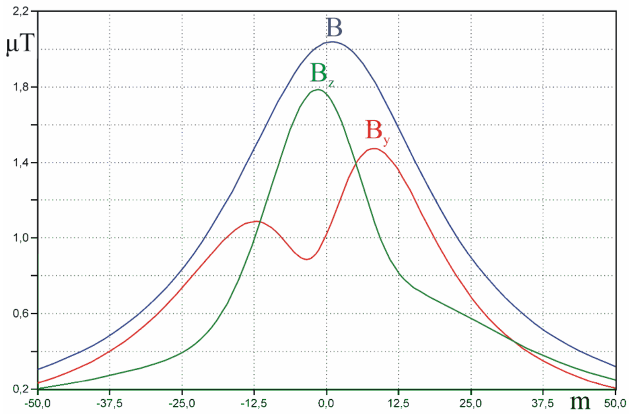

Fig. 3. Effective values of the components and the total magnetic flux density of the overhead power line.

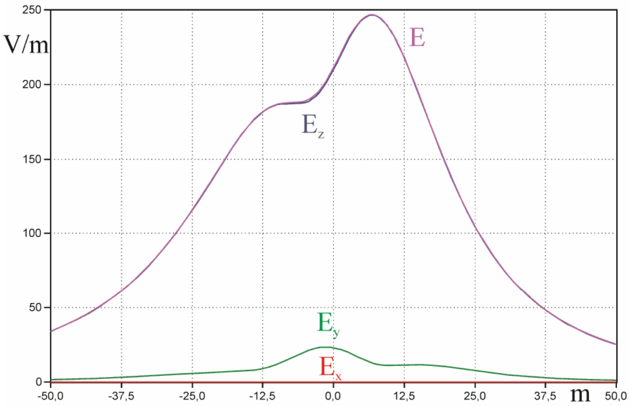

Fig. 4. Effective values of the components and the total electric field intensity of the overhead power line.

B. Buried cable line

A 35 kV buried cable line, length $\ell = 1$ km, consists of three phase conductors situated along the *x*-axis. The input data and coordinates are given in Table 2. Power frequency $f = 50$ Hz.

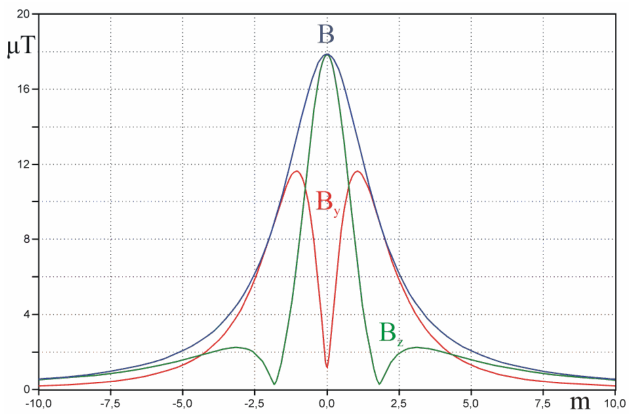

Fig. 5. Effective values of the components and the total magnetic flux density of the buried cable line.

Fig. 5 presents effective values of components and the total effective value of the magnetic flux density along the y-directed observation profile at 1 m above the earth surface. The total electric field intensity of this buried cable line, along the same profile, is shown in Fig. 6.

TABLE 2: INPUT DATA FOR BURIED CABLE POWER LINE.

	i	*y(m)*	*z(m)*	$\bar{I}^{\ell}$ *(A)*
L1	1	0	-0.8	$805\angle-30°$
L2	2	-0.21	-0.8	$805\angle-150°$
L3	3	0.21	-0.8	$805\angle 90°$

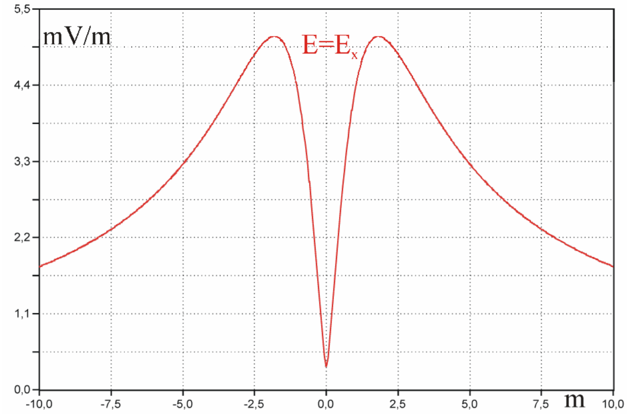

Fig. 6. Total effective value of the electric field intensity of the buried cable line.

VII. CONCLUSION

Here presented numerical algorithm for the computation of the power frequency electromagnetic field of overhead and buried power lines is based on the separation of the phase conductor currents into their longitudinal and transversal components. The magnetic and electric fields computations are based on the Biot-Savart law and the average potential method.

This algorithm is suited for computation of electric and magnetic field of the relatively short straight sections of power lines. In the case of very long power line, results obtained by the algorithm developed in this paper are in good agreement with those obtained by the algorithm developed for infinitely long straight power line.

REFERENCES

[1] *ICNIRP Guidelines for Limiting Exposure to Time-Varying Electric, Magnetic and Electromagnetic Fields (up to 300 Hz)*, Health Physics, vol. 74, no. 4, pp. 494-522, 1998.

[2] Z. Haznadar, Ž. Štih, *Electromagnetic Fields, Waves and Numerical Methods*, Amsterdam: IOS Press, 2000.

[3] S. Vujević, P. Sarajčev, and A. Botica, "Computation of the overhead power line electromagnetic field," in *16th International Conference on Software, Telecommunications and Computer Networks (SoftCOM 2008),* Split, 2008, pp. 27-31.

[4] P. Sarajčev, "Electromagnetic model of the system of conductors in multilayer medium," (in Croatian), Ph.D. Thesis, University of Split, FESB, Split, 2008.

[5] J. Moore and R. Pizer (editors), *Moment Methods in Electromagnetics - Techniques and Applications*, New York: John Wiley & Sons, 1984.

[6] S. Vujević, "Time-Harmonic Analysis of Earthing Grids," in *Electrical Engineering and Electromagnetics VI*, C. A. Brebbia and D. Poljak, Ed. Southampton, Boston: WIT Press, 2003, pp. 235-244.

[7] D. L. Garrett and J. G. Pruitt, "Problems encountered with the average potential method of analyzing substation grounding systems," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-104, no. 12, pp. 3585-3596, 1985.

[8] S. Vujević and P. Sarajčev, "Potential distribution for a harmonic current point source in horizontally stratified multilayer medium," *COMPEL*, vol. 27, no. 3, pp. 624-637, 2008.