Stochastic Characterization of Amplified Photons in Lightwave Systems with Optically Bistable Elements

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***Abstract* — An optical amplifier followed by a bistable element is statistically analyzed. To this end, the birth-death-immigration model and the coupled mode theory are used to characterize the amplifier and the bistable element, respectively.**

***Keywords* — Optical Amplifier, Optical Bistability, Probability Density Function, Statistical Analysis.**

I. Introduction

NONLINEAR elements are very much needed in all-optical lightwave systems where the extremely high data rate information is to be all-optically processed. In particular, optical bistable elements have recently been shown to be useful as photonic thresholders in optical communication systems [1]. However, inasmuch as optical bistability- much like other optical nonlinearities- cannot be achieved but at high power levels [2-3], optical amplifiers should be inevitably used in the system [4-5], where the received photons are amplified before being fed to the bistable device. In this manuscript; therefore, a stochastic characterization of amplified photons in lightwave systems with optically bistable elements is given for the first time. It is shown that optical bistable thresholders could be still helpful elements even at the expense of using optical amplifiers.

The structure of this manuscript is as follows: the birth-death-immigration (BDI) model is first employed to characterize the statistics of amplified photons. The coupled mode theory (CMT) is then applied to analytically model the bistable thresholder which could be a photonic crystal based direct-coupled structure [6]. Finally, the photon statistics of the output is numerically studied and the noise figure of the nonlinear receiver with optically bistable thresholder is presented.

II. Statistics of Amplified Photons in Bistable Elements

The statistical characterization of the optical bistable element being fed with the optically amplified photons is performed at two steps.

First, the BDI model is employed to characterize the overall performance of the optical amplifier schematically shown in Fig. 1. Since the received photons are assumed to be coherent and Poisson distributed (the corresponding photon-number is denoted by N_{Pois} in the figure), it can be shown that the amplified photons (the corresponding photon-number is denoted by N_{amp} in the figure) are Laguerre distributed. The amplified photon-number mean; $\langle N_{amp}\rangle$, and variance; Var_{amp}, then read as [7]:

$$\langle N_{amp}\rangle = G\,\langle n_s\rangle + M\,\langle n_a\rangle \quad (1)$$

$$Var_{amp} = G\,\langle n_s\rangle + 2G\,\langle n_s\rangle\langle n_a\rangle + M\,\langle n_a\rangle\left(1+\langle n_a\rangle\right) \quad (2)$$

where G is the gain of amplifier, M is number of spontaneous emission modes, $\langle n_s\rangle$ is the mean value of the amplifier input, $\langle n_a\rangle = n_{sp}(G\text{-}1)$, and n_{sp} is the spontaneous emission factor of the amplifier [8].

Second, the amplified photons are fed to the optical bistable element schematically shown in Fig. 1. The output photon-number; thanks to the CMT, can be analytically expressed as:

$$\frac{N_{out}}{N_{in}} = \frac{1}{1+\left(N_{out}/N_{ch}-\delta\right)^2} \quad (3)$$

where δ is referred to as the normalized detuning factor of the carrier frequency, and N_{ch} is the characteristic number of photons in the device [6].

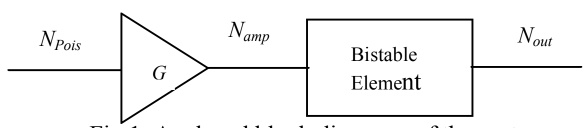

Fig.1. Analyzed block diagrams of the system

The output photon-numbers normalized to N_{ch} can then be written down in terms of the input photon-numbers normalized to N_{ch}, i.e. amplified photon-numbers:

$$N_{out}^{\,3} - 2\delta N_{out}^{\,2} + (1+\delta^2)N_{out} - N_{amp} = 0 \quad (4)$$

In accordance with Fig. 2; however, two different scenarios are possible for a bistable element. Depending on whether the amplified photon-number level $N_{in}=N_{amp}$ belongs to regions I/III or to region II, the output photon-number solved from Eq.(4) could be either single-valued or triple-valued. These regions could be easily identified by using the here proposed Δ factor:

$$\Delta = \frac{q^2}{4} + \left(\frac{p}{3}\right)^3 \quad (5)$$

where:

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$$q = 2\left(\frac{\delta}{3}\right)^3 + 2\left(\frac{\delta}{3}\right) - N_{amp} \quad (6)$$

and p=1-3(δ/3)^2 .We are in region I/III for $\Delta > 0$ and:

$$N_{out} = 2\left(\frac{\delta}{3}\right) + \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \left(\frac{p}{3}\right)^3}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \left(\frac{p}{3}\right)^3}} \quad (7)$$

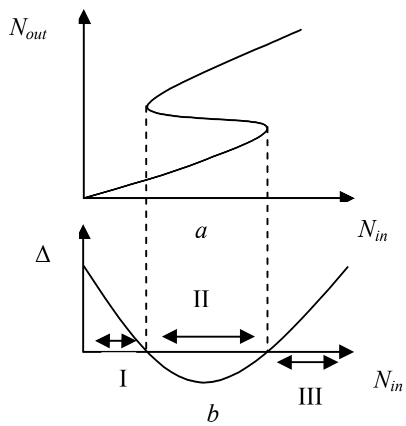

Fig.2. a) Typical O.B. characteristic; b) Δ versus N_{in} and related three regions

For $\Delta < 0$; on the other hand, we are in region II, and there are three different possibilities for the output photon-number value:

$$\begin{cases} N_{out} = 2\left(\frac{\delta}{3}\right) + 2\sqrt{-\frac{p}{3}}\cos\left(\frac{\alpha}{3}\right) \\ N_{out} = 2\left(\frac{\delta}{3}\right) + 2\sqrt{-\frac{p}{3}}\cos\left(\frac{\alpha}{3} + \frac{2\pi}{3}\right) \\ N_{out} = 2\left(\frac{\delta}{3}\right) + 2\sqrt{-\frac{p}{3}}\cos\left(\frac{\alpha}{3} + \frac{4\pi}{3}\right) \end{cases} \quad (8)$$

where $\alpha = Cos^{-1}(-q/2\sqrt{(-p/3)^3})$.Although there are three different solutions for the output photon-number level given in equations (8), only the maximum and the minimum values corresponding to the stable branches of the bistable diagram are acceptable.

Now that the output photon-numbers are analytically determined in terms of amplified photon-numbers, the probability density function (PDF) of output photons, N_{out}, can be found in terms of $P_N(N_{amp})$, the PDF of amplified photon-numbers.

$$P_{N_{out}}(N_{out}) = \begin{cases} \left(3N_{out}^2 - 4\delta N_{out} + (1+\delta^2)\right) P_N(N_{amp}) & \text{for regions I,III} \\ \frac{1}{2}\left(3N_{out}^2 - 4\delta N_{out} + (1+\delta^2)\right) P_N(N_{amp}) & \text{for region II} \end{cases} \quad (9)$$

It should be noticed that $P_N(N_{amp})$ is a Laguerre distribution whose mean and variance are given in Eqs. (1-2) [4]. It is now possible to extract the desired statistical characteristic of the output photon-numbers.

III. SIMULATION AND NUMERICAL RESULTS

As a numerical example, an optical amplifier with $\langle n_s \rangle$=36, G=10, $\langle n_a \rangle$=3 and M=5 followed by a bistable element with N_{ch}=240 and δ=2 is considered. The PDF of the amplified photon-numbers, $P_N(N_{amp})$, and that of the output photon-numbers, $P(N_{out})$, are plotted in Fig. 3. This figure clearly demonstrates the bistability of output photon-numbers.

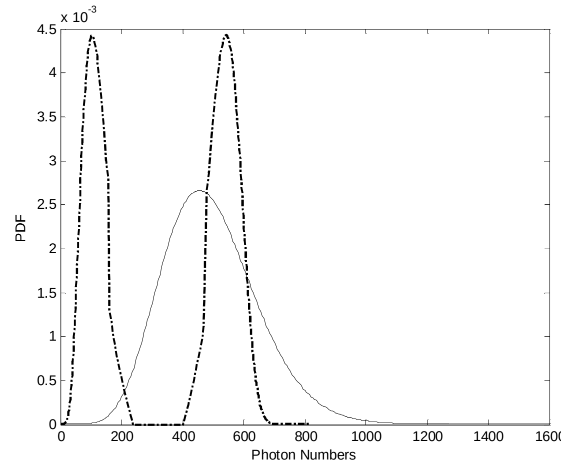

Fig.3. The PDF of the amplified photon-numbers (solid line), The PDF of the output photon-numbers (dashed line).

IV. CONCLUSIONS

A list In this manuscript, the photon-number statistics of amplified photons fed into optically bistable elements are given. The BDI model is employed to characterize the statistics of the amplifier and the CMT is applied to analytically model the bistable elements. In this fashion, the PDF of output photon-numbers is analytically calculated and used to extract the mean and variance of the output photon-numbers. The obtained results indicates that the stochastic characteristics of the output photons is a random variable whose statistical parameters can be numerically related to the amplification parameters (G, n_a, M), and bistability features (N_{ch}, δ).

REFERENCES

[1] H. Abediasl et. al., "Performance analysis of optically bistable thresholders in optical code division multiple access networks," *Proc. SPIE*, vol. 6729,pp. 67292C, 2007.
[2] M. Soljačić and J. D. Joannopoulos, "Enhancement of nonlinear effects using photonic crystals," *Nat. Mater.*, vol. 3, no. 4, pp. 211–219,Apr. 2004.
[3] K. Sakoda and K. Ohtaka, "Sum-frequency generation in a two-dimensional photonic lattice," *Phys. Rev. B, Condens. Matter*, vol. 54,no. 8, pp. 5742–5749, Aug. 1996.
[4] M. Razavi, J. A. Salehi, "Statistical analysis of fiber-optic CDMA communication systems. I. Device modeling," Journal *of Lightwave Technology*, vol. 20, no. 9,pp. 1304, Aug 2002.
[5] M. J. Connelly, "Wideband semiconductor optical amplifier steady-state numericalmodel," *Quantum Electronics, IEEE Journal of*, vol. 37,pp. 439-447,Mar 2001.
[6] J. D. Joannopoulos, et. al., "Modeling nonlinear optical phenomena in nanophotonics," *Journal of Lightwave Technology*, vol. 25, no. 9,pp. 2539-2546, Sep 2007.
[7] S. Zahedi, J. A. Salehi, "Analytical comparison of various fiber-optic CDMA receiverstructures", *Journal of Lightwave Technology*, vol. 18, no. 12,pp. 1718-1727, Dec 2000.
[8] R. M. Gagliardi, " Optical communications ," *John Wiley*,pp. 102-106, 1995.