

Developing Frequency Response Analyzer in MATLAB[®] Simulink Environment

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Abstract — The paper describes the application of the correlation technique in the linear system frequency response analysis. An example of the system with non-rational transfer function is used to demonstrate the merits of the developed frequency response analyzer implemented in MATLAB[®] Simulink environment.

Keywords — Frequency response analysis, zero-order hold, PADÉ approximation, MATLAB[®] Simulink.

I. INTRODUCTION

FREQUENCY response function measurements are an interesting intermediate step in the identification process with nonparametric results. However, there are methods in the literature [1] that provide parametric models from available nonparametric data.

Frequency response analysis offers some useful insights into stability and other characteristics of the control system. Frequency response allows us to understand the system behavior in the presence of more complex inputs. However, the frequency response methods, may be less intuitive than other design methods. The main disadvantage of frequency response analysis is that many industrial processes do not admit sinusoidal inputs during normal operation, especially because experiment repeating for many frequencies may lead to long research period.

The layout of the paper is as follows: in Section 2 we give a brief review of the frequency response analysis based on the using the correlation method; Section 3 addresses to MATLAB[®] Simulink implementation of the considered frequency response analyzer. Section 4 contains an example of using the developed analyzer.

II. FREQUENCY RESPONSE ANALYSIS

A. The Frequency Response of Continuous-Time Linear Time-Invariant Systems

Consider a continuous-time, linear time-invariant (LTI)

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system between the input $u(t)$ and the output $y(t)$ as shown in Fig. 1. The function $G(j\omega)$ is called the frequency response of the system, where $U(j\omega)$, $Y(j\omega)$, and $G(j\omega)$ are the Fourier transform of input $u(t)$, output $y(t)$, and impulse response $g(t)$, respectively.

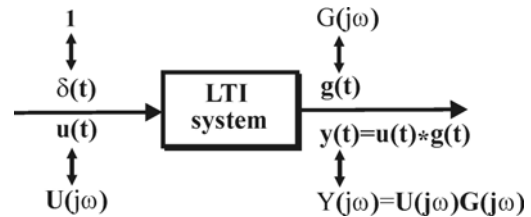


Fig. 1. Relationships between inputs and outputs in a LTI system.

The frequency response analysis is a simple method for obtaining detailed information about the considered linear system. Recall that in the case of a mono-frequent sinusoidal input signal

$$u(t) = a \sin(\omega t), \quad (1)$$

the uncorrupted, steady-state response of a linear, stable system with transfer function $G(s)$ is

$$y(t) = b \sin(\omega t + \varphi), \quad (2)$$

whose evaluation is shown in Fig. 2(a). By measuring the signals with an oscilloscope, the amplitude and phase of the system frequency response can be promptly computed giving

$$|G(j\omega)| = \frac{b}{a}, \quad \varphi(\omega) = \angle G(j\omega) = -\omega t_\varphi = -\frac{t_\varphi}{T_0} 360^\circ, \quad (3)$$

where T_0 and t_φ are the signal period and the delay time, respectively. This experiment should be repeated for a number of frequencies in a certain frequency band. As shown in Figs. 2(d) and (e), the frequency response data can be visualized in two different ways: via the NYQUIST diagram or via the BODE plots.

B. Frequency Response Analysis by the Correlation Method

The problems of non-linear distortion, as well as noise corruption of the output signals, that usually occur in engineering systems (see Fig. 2(c)), can be overcome in the measurement scheme given in Fig. 3, by applying time averages. This set-up visualizes a correlation method in order to see how much energy is at the frequency in question [2]. Briefly, the correlation frequency analyzer functions as follows [3], [4].

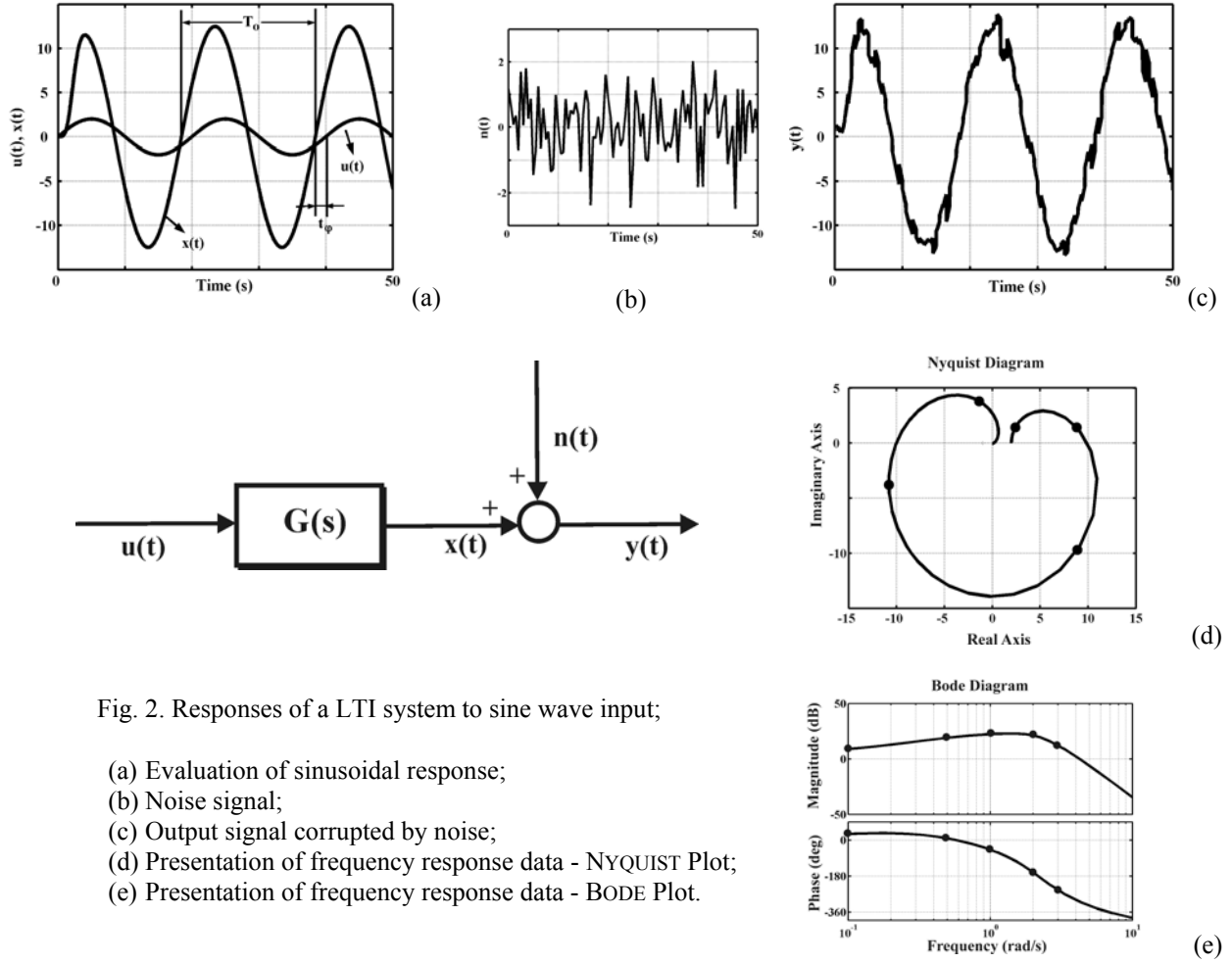


Fig. 2. Responses of a LTI system to sine wave input;

- (a) Evaluation of sinusoidal response;
- (b) Noise signal;
- (c) Output signal corrupted by noise;
- (d) Presentation of frequency response data - NYQUIST Plot;
- (e) Presentation of frequency response data - BODE Plot.

The measured system output is multiplied with a sine and cosine signal of the frequency of the system input ω . The products are then integrated for a specified measurement time T_m . As the averaging time increases, the contribution of all unwanted frequency components in $y(t)$ goes to zero, and the integrator outputs ($y_s(T_m), y_c(T_m)$) become constant values that depend only on the gain and phase of the considered system transfer function at the test frequency.

For specific values of integration time, the sine channel output $y_s(T_m)$, given by

$$\begin{aligned}
 y_s(T_m) &= \int_0^{T_m} y(t) \sin(\omega t) dt = \\
 &= \int_0^{T_m} b \sin(\omega t + \varphi) \sin(\omega t) dt + \int_0^{T_m} n(t) \sin(\omega t) dt,
 \end{aligned} \tag{4}$$

represents a measurement of the real part of the system frequency response at the test frequency ω . Namely, the previous equation (4) can be rewritten in the form

$$\begin{aligned}
 y_s(T_m) &= \\
 &= \underbrace{\frac{b}{2} T_m \cos \varphi}_{1^{st} \text{ term}} - \underbrace{\frac{b}{2} \int_0^{T_m} \cos(2\omega t + \varphi) dt}_{2^{nd} \text{ term}} + \underbrace{\int_0^{T_m} n(t) \sin(\omega t) dt}_{3^{rd} \text{ term}}.
 \end{aligned} \tag{5}$$

Note that in particular for $T_m = \frac{kT_0}{2}$, $k = 1, 2, 3, \dots$, the second term in Eq. (5) approaches zero. Also, it can be shown that the average of the integrated noise (the third term in Eq. (5)) is zero, which implies that, selecting the sufficient number of test signal periods, the desired accuracy can be reached even in case of low signal-to-noise ratio [3]. Therefore, the sine channel output in Fig. 3 becomes

$$y_s(T_m) \approx \frac{bT_m}{2} \cos \varphi. \tag{6}$$

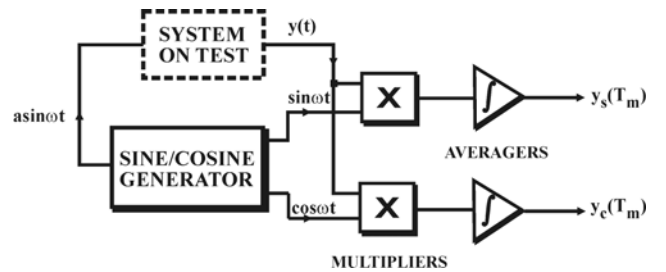


Fig. 3. Block-diagram of the correlation frequency response analyzer.

Similarly, the cosine channel output $y_c(T_m)$ can be evaluated for $T_m = \frac{kT_0}{2}$, $k = 1, 2, 3, \dots$ as

$$y_c(T_m) \approx \frac{bT_m}{2} \sin \varphi. \quad (7)$$

Finally, the gain and phase of the system frequency response can be calculated using the following equations:

$$|G(j\omega)| = \frac{2}{aT_m} \sqrt{y_s^2(T_m) + y_c^2(T_m)}, \quad (8)$$

and

$$\angle G(j\omega) = \arctg \frac{y_s(T_m)}{y_c(T_m)}.$$

Note that the four quadrant inverse tangent function has to be used.

III. MATLAB[®] SIMULINK IMPLEMENTATION OF THE CORRELATION FREQUENCY RESPONSE ANALYZER

The method described in the previous section can easily be interpreted as determination of the basic component of a FOURIER series expansion of the considered process output. In fact, all modern transfer function analyzers are organized as FOURIER analyzers, that process signals sampled at equidistant points of time by digital hardware.

Since the standard Simulink block library does not provide any Frequency Response Analyzer (FRA) block, the above described correlation method is implemented in MATLAB[®] Simulink, performing all necessary calculations automatically for each value specified in the angular frequency vector.

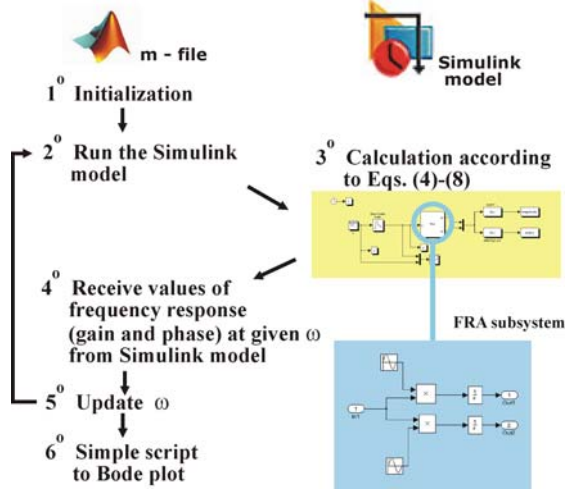


Fig. 4. Schematic for the implementation of FRA.

Note that to define the frequency interval, where it is essential to know the process, some a priori information is necessary. In determining the amplitude of the input signal $u(t)$, the linearity of the system should be preserved. Also, a sufficient long measurement time T_m , chosen as a multiple of full periods of the test signal, implies desired accuracy and low noise sensitivity.

The schema of the implementation is shown in Fig. 4. A major part is implemented in Simulink, whereby the created mdl-file is run by an m-file that is provided for initialization, frequency updating, Bode plotting, etc.

To verify that the program works properly, the implemented method can be used in case of a linear time-

invariant system with known transfer function, and compare the obtained characteristics with those obtained using MATLAB[®] in-built function `bode`.

IV. AN EXAMPLE OF USING THE DEVELOPED ANALYZER

In the previous section explored frequency response analyzer can be verified on the example of the zero-order hold device whose dynamic behavior can be described by the non-rational transfer function.

Recall that in sampled-data control systems hold circuits are used to convert the discrete-time signals from digital compensators into the continuous-time signals to be applied to the continuous-time objects. Hold circuits can be viewed also as filters which attenuate the high frequency alias spectra generated by sampling continuous-time signals.

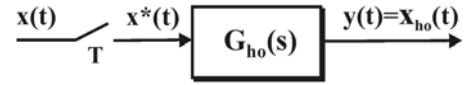


Fig. 5. The block-diagram of the idealized model of the sample-and-zero-order hold circuit

As Fig. 5 visualizes, the idealized model of a sample-and-zero-order hold (ZOH) circuit is obtained by combining a sampler, described by using impulse modulation

$$x^*(t) = x(t)i(t) = \sum_{k=-\infty}^{\infty} x(t) \cdot \delta(t - kT), \quad (9)$$

and a zero-order-hold circuit, given by transfer function

$$G_{h0}(s) = \frac{1 - e^{-sT}}{s}, \quad T - \text{is sampling period}. \quad (10)$$

A. Rational approximation of $G_{h0}(s)$

Since the transfer function (10) is not a rational function, it is suitable to find a finite-dimensional rational approximation. Among the many methods, PADÉ approximations are the most frequently used methods to approximate by a rational function the dead-time τ , given in continuous-time control systems with transfer function $e^{-s\tau}$. The standard PADÉ approximation (recommended in many textbooks) with equal degrees of the polynomials in numerator and denominator, exhibits a jump at time $t = 0$. To avoid this phenomenon, which is undesirable in dead-time simulating, the PADÉ approximation, where the numerator degree is one less than the denominator degree, is recommended. Hence, the PADÉ approximation of order (m, n) is defined to be a rational function $R_{m,n}(s)$ expressed in a fractional form [5]:

$$e^{-\tau s} \approx R_{m,n}(\tau s) = \frac{N_m(\tau s)}{D_n(\tau s)} \quad (11)$$

$$N_m(\tau s) = \sum_{k=0}^m (-1)^k \frac{(m+n-k)!m!}{(m+n)!k!(m-k)!} \tau^k s^k \quad (12)$$

$$D_n(\tau s) = \sum_{k=0}^n \frac{(m+n-k)!n!}{(m+n)!k!(n-k)!} \tau^k s^k \quad (13)$$

Notice that $R_{m,0}(\tau s)$ is the m -th order TAYLOR

polynomial. High-order PADÉ approximations produce transfer functions with clustered poles. Because such pole configurations tend to be very sensitive to perturbations, PADÉ approximations with order $n > 10$ should be avoided. The approximation $G_{9,10}(s) = [1 - R_{9,10}(Ts)]/s$ for the transfer function $G_{h0}(s)$ is adopted in this paper.

In the case of a relative low sampling frequency of 10 samples per cycle of the sinusoidal input $x(t)$, Fig. 6 visualizes the output of the sampler/zero-order hold $x_{h0}(t)$, as well as the response of the function $G_{9,10}(s)$ corrected by T according to sampling theory.

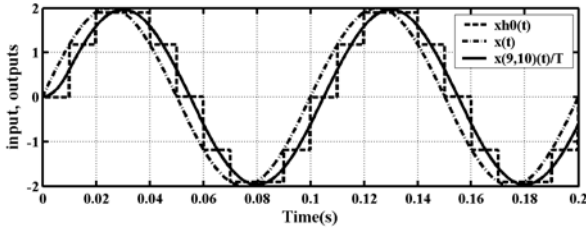


Fig. 6. Responses of ZOH to sine wave input.

B. Frequency responses –Results and Discussion

To obtain the frequency responses (magnitude and phase) of the zero-order hold, consider the following expressions:

$$|G_{h0}(j\omega)| = \frac{2\pi}{\omega_s} \left| \text{sinc} \left(\pi \frac{\omega}{\omega_s} \right) \right|, \quad \omega_s = \frac{2\pi}{T},$$

and

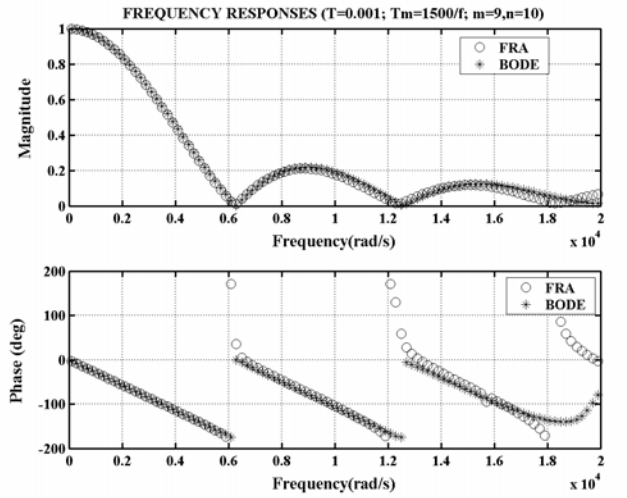
$$\arg G_{h0}(j\omega) = -\pi \frac{\omega}{\omega_s} + \theta, \quad \theta = \begin{cases} 0 & \sin(\pi\omega/\omega_s) \geq 0 \\ \pi & \sin(\pi\omega/\omega_s) < 0 \end{cases}$$

Some results of frequency response analysis are presented in the form of the BODE diagrams, which are given for two different values of sampling period $T = 0.001$ s and $T = 0.01$ s in Figs. 7(a) and 7(b), respectively. The curves denoted by 'circle' symbol represent the frequency responses of ZOH device determined by the considered correlation technique, while the curves denoted by 'star' symbol are the BODE responses of the rational approximation of order $(m, n) = (9, 10)$ of the transfer function $G_{h0}(s)$ according (10)-(13). It can be concluded that the correlation technique provides a quite fine estimation of the frequency responses, with good fits to the amplitude and phase of $G_{h0}(j\omega)$ given by (14). At higher frequencies the responses (the phase responses, particularly) are distorted, which can be mainly attributed to approximation errors in Eqs. (6), (7) and (11).

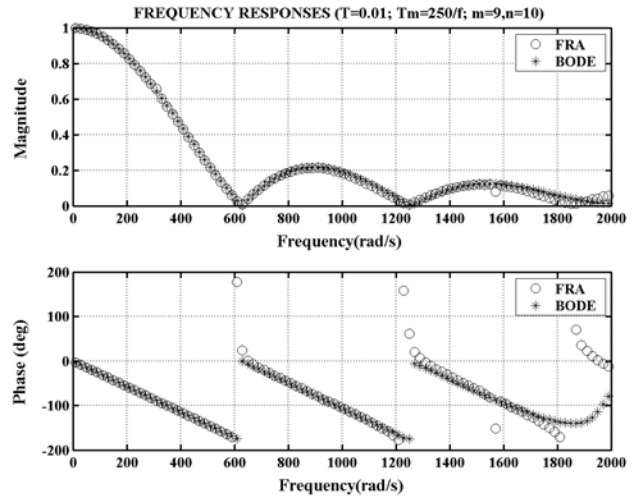
V. CONCLUSION

The paper presents a frequency response analyzer implemented in MATLAB[®] Simulink environment. This work suggests a number of improvements for future investigation. Bearing in mind that the ultimate goal of this research is a frequency response analyzer that works in real time experiment [6], its full Simulink

implementation should be explored. Also, it would be suitable to consider the possibility of generating only one appropriate chirp test signal for the specified frequency range, instead a large number of sine wave signals at different test frequencies.



(a)



(b)

Fig. 7. The frequency responses of zero-order hold for different sampling time: (a) $T = 0.001$ s; (b) $T = 0.01$ s.

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