Control Design of a Positioning System upon a Fault Tolerant Multisensor Scheme

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***Abstract*— The present paper deals with a fault tolerant control scheme for a multisensor plant under the assumption of bounded noises. A practical example, concerning a positioning system is detailed. Robust guarantees for the global stability of the system and the separability and identification of abrupt faults occurring in the sensor outputs are provided.**

I. INTRODUCTION

As it is usually the case with the diversification and miniaturisation with low cost solutions, components are predisposed to failures. For multisensor schemes, the presence of faults is manifested by the alteration of the estimations of the features of interest. The control strategy has to be equipped with fault detection capabilities in order to avoid the construction of the control action based upon erroneous feedback information.

Multisensor schemes have originated substantial research on the aggregation of the information available from the plant in order to improve reliability and robustness. Sensor fusion has been one of the techniques traditionally employed in multisensor schemes where the construction of improved estimators is the main concern [1], [2], [3].

By contrast, robust fault diagnosis procedures are less often found in literature. An example is [4] and, more recently, multisensor switching feedback control strategies with fault tolerance guarantees were presented in [5].

In the present paper, a multisensor scheme, similar in design with the one detailed in [5] will be implemented upon the practical example of a positioning system. A FDI (Fault Detection and Isolation) mechanism which assures the robust selection of healthy sensors for the feedback loop design will be used. Set membership techniques, based on the invariant sets studied in [6] will be used.

The following notations will be used throughout the paper. $\mathbb{N}$ denotes the set of nonnegative integers; $\mathbb{N}^+$ denotes the set $\mathbb{N}\setminus\{0\}$. Whenever time is unspecified, a variable x stands for $x(k)$ for some (unspecified) $k \in \mathbb{N}$, and x^+ stands for the *successor* variable, i.e. $x(k+1)$. The Minkowski sum of two sets is defined as $A \oplus B = \{a+b : a \in A \text{ and } b \in B\}$.

The class of polyhedral sets understood as intersections of a finite number of half spaces will be used extensively in this paper. A polytope is a closed and bounded polyhedra.

The remainder of the paper is organised as it follows. Section II introduces the multisensor control structure. Section III details the invariant sets required in the fault tolerant scheme, Section IV describes the fault tolerant switching control scheme. In Section V an application is presented and Section VI draws some conclusions.

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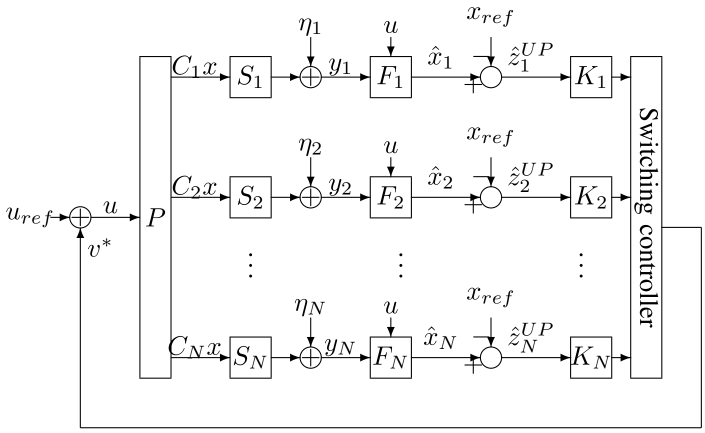

Fig. 1: Multisensor control scheme

II. PLANT DYNAMICS AND FAULT SCENARIO

The multisensor control scheme considered in the present paper is a linear discrete-time state space model of the plant:

$$x^+ = Ax + Bu + Ew \tag{1}$$

where $x \in \mathbb{R}^n$ and $x^+ \in \mathbb{R}^n$ are, respectively, the current and successor system states, $u \in \mathbb{R}^m$ is the input, and $w \in W \subset \mathbb{R}^r$ is a bounded process disturbance. Matrix A is assumed to be invertible (this is always the case if system (1) corresponds to the exact discretization of an underlying continuous-time system) and the pair (A, B) is controllable.

The control objective is for the state of the plant (1) to track a reference signal x_{ref} that satisfies

$$x_{ref}^+ = Ax_{ref} + Bu_{ref} \tag{2}$$

The state reference is considered to be bounded by the closed polyhedral set $X_{ref} \subset \mathbf{R}^n$.

We will use a multisensor switching scheme with plant P, sensors $S_1, \ldots, S_N$, estimators $F_1, \ldots, F_N$ (see Figure 1).

A. *Sensor and estimator dynamics*

The state vector x is not directly measurable, but linear combinations of it, $C_i x, \quad i = 1, \ldots, N$ can be measured via N sensors. The sensors are considered to have no dynamics and their output signal is:

$$y_i = C_i x + \eta_i \tag{3}$$

The sensor faults considered in this paper are considered to be abrupt, that is the fault manifests itself in the fact that the output no longer carries information about the sensor state. The observation equation thus, becomes:

$$y_i = \eta_i^F \tag{4}$$

The noise occurring during the fault, $\eta_i^F \in \mathbf{N_i^F} \subset \mathbf{R^p}$, may be different from the one during the healthy functioning, $\eta_i \in \mathbf{N_i} \subset \mathbf{R^p}$. Without any loss of generality the bounded sets of disturbances w, η_i and η_i^F, for $i = 1, \ldots, N$ are considered to be bounding boxes.

The estimators are designed such that they will have an adequate dynamic behaviour for the plant state estimate:

$$\hat{x}_i^+ = \underbrace{(A - L_i C_i)}_{A_{L_i}} \hat{x}_i + Bu + L_i \eta_i \tag{5}$$

with the gains L_i chosen such that matrices A_{L_i} are strictly stable (always possible by the detectability assumption).

Using (1) and (5) one can define the estimation errors affecting the sensor:

$$\tilde{x}_i^+ = x^+ - \hat{x}_i^+ = A_{L_i} \tilde{x}_i + \begin{bmatrix} E & -L_i \end{bmatrix} \begin{bmatrix} w \\ \eta_i \end{bmatrix} \tag{6}$$

The tracking errors are given by the difference between the state and their respective reference signal:

$$z^+ = x^+ - x_{ref}^+ = Ax + B \underbrace{(u - u_{ref})}_{v} + Ew \tag{7}$$

To alleviate sharp changes in the value of the reference signal, the update estimations are provided:

$$\hat{x}_i^{UP} = \hat{x}_i + M_i \left(y_i - C_{s_i} \hat{\xi}_i \right)$$

with matrices M_i determined from equation

$$A_{L_i} M_i = L_i \tag{8}$$

B. Closed loop dynamics

The fault tolerant scheme works under the assumption that only healthy sensors will be used in the control law design.

Through a FDI mechanism, detailed in Subsection IV we are able to identify and select only the healthy, (understood as a sensor with a healthy functioning in the sense of (3) and for which the estimation error (6) is confined in a safety region) sensors and from them, an index can be chosen as the minimiser of a given cost function:

$$\hat{z}^* = \min_{i \in \mathcal{I_H}} J \left(\hat{z}_i^{UP} \right) \tag{9}$$

further, the control action has the form:

$$u = u_{ref} + v^* = u_{ref} - K_l \hat{z}^* \tag{10}$$

The feedback gain can be, for example, computed as the solution to a Riccati equations for a given set of tuning parameters.

As a consequence of the selection of a healthy sensor, by using (3), (7) and (8), we have

$$\hat{z}_l^{UP} = z - (I - M_l C_l) \tilde{x}_l + M_l \eta_l \tag{11}$$

and, the control action (10) can be expressed as

$$u = u_{ref} - K \hat{z}_l^{UP} = u_{ref} - BK \left(z - (I - M_l C_l) \tilde{x}_l + M_l \eta_l \right) \tag{12}$$

III. INVARIANT SETS

The fault tolerant scheme implemented requires the use of invariant sets. Usually, such a construction may prove computationally difficult [7]. In this paper, a useful construction, detailed in [6] and extensively used in [5], will be used to obtain RPI (robust positively invariant) sets for the signals of interest. In the following the sets corresponding to the estimation error (6) and plant tracking error (7) will be constructed.

A. Estimation error

For the dynamics (6) one can obtain, for $i = 1, \ldots, N$,

$$\tilde{S}_i(\epsilon) = \Big\{ \tilde{x}_i \in \mathbf{R}^n : \left| V_i^{-1} \tilde{x}_i \right| \leq (I - |\Lambda_i|)^{-1} \left| V_i^{-1} \begin{bmatrix} E & -L_i \end{bmatrix} \right| \begin{bmatrix} \bar{w} \\ \bar{\eta}_i \end{bmatrix} + \epsilon \Big\} \tag{13}$$

with Λ_i and V_i given by the Jordan decomposition $A - L_i C_i = V_i \Lambda_i V_i^{-1}$, and $\bar{w}$ and $\bar{\eta}_i$ bounds for the sets of uncertainties, respectively, W and N_i.

B. Plant tracking error

Using (1), (2), (6), (7) and (12) we have:

$$z^+ = A_{z,l} z + B_{z,l} \begin{bmatrix} w \\ \tilde{x}_l \\ \eta_l \end{bmatrix} \tag{14}$$

with $A_{z,l} = A - B_l K_l$, $B_{z,l} = \begin{bmatrix} E & B_l K_l (I - M_l C_l) & B_l K_l M_l \end{bmatrix}$. For dynamics (14) one can obtain

$$S_z(\epsilon) = \Big\{ z \in \mathbf{R}^n : \left| V_{z,l}^{-1} z \right| \leq (I - |\Lambda_{z,l}|)^{-1} \left| V_{z,l}^{-1} B_{z,l} \right| \begin{bmatrix} \bar{w} \\ \bar{\bar{x}}_l \bar{\eta}_l \end{bmatrix} + \epsilon \Big\} \tag{15}$$

with $\Lambda_{z,l}$ and $V_{z,l}$ given by the Jordan decomposition $A_{z,l} = V_{z,l} \Lambda_{z,l} V_{z,l}^{-1}$, and $\bar{w}$, $\bar{\bar{x}}_l$ and $\bar{\eta}_l$ bounds for the sets of uncertainties, respectively, W, the bounding box maximum of set (13) and N_l.

IV. FAULT TOLERANT SCHEME

A. Separation

From the classical fault detection and isolation point of view ([8]), a signal called a residual, sensitive to fault occurrences and with a manageable dependence on the disturbances can be defined for the detection of faults. Indeed, the presence of faults implies a modification in the dynamic equation of the corresponding estimator which thus carries information on the fault signature.

As such, the residual signal

$$r_i = \hat{z}_i^{UP} - (I - M_i C_i) \hat{z}_i \tag{16}$$

composed from all the measurable quantities associated to the i^{th} sensor can be defined, and from (3) and (4) we find that the sets inside which resides, for the healthy, respectively the faulty cases are

$$\begin{aligned} S_i^H &= M_i C_i S_z \oplus M_i N_i \\ S_i^F &= \{-M_i C_i S_{x_{ref}}\} \oplus M_i N_i^F \end{aligned} \quad (17)$$

In the rest of the paper, an explicit separation will be used in order to guarantee the separation. As it can be seen in Figure 2 there are cases in which the separation must be considered pair by pair, for each sensor in order to ascertain the sensor state. At each instant of time the residual signal (16) of a sensor will be verified against the sets (17). If the sensor is deemed to be faulty, it will be discarded from the sets of selectable sensors. It will no longer be used in the construction of the command law.

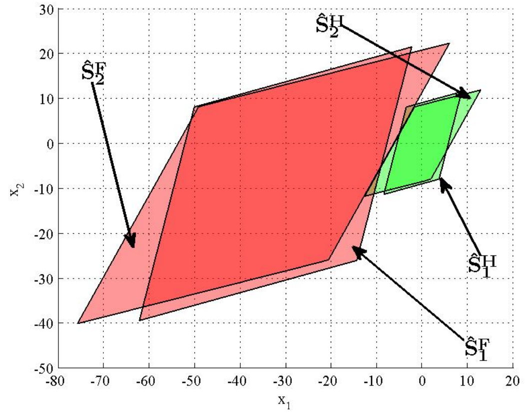

Fig. 2: Sensor separation

One can remark that at each moment of time only one sensor is selected into the design of the command law thus discarding the redundant information provided by the other healthy sensors. This contrasts with fusion schemes which use stochastic techniques in combining the information of all the available sensors. The advantage of the robust scheme presented here resides in that the information provided by a faulty sensor will never be used by the command law.

V. Practical Example

In the following, a position control plant will be tested under the fault tolerant scheme. The relevant parameters of the system will be determined and then an example of functioning will be presented.

A. System identification

The goal is for a linear cursor to follow a given electrical reference signal. The cursor is attached to a belt moved by a continuous current engine through a pulley and a reducer. The pulley transforms the rotation into a linear movement of the electrical engine and the reducer facilitates the obtaining of a better precision of the cursor position through a reduced inertia of the motor axis.

The practical assembly has two sensors. A position sensor which measures the linear position of the cursor and a tachometric generator which gives a proportional tension with the rotation speed of the motor.

The above elements are presented in Figure 1 where the plant with the sensors is presented in open-loop. The offset value is added to counter the influences of the operational amplifiers used in the scheme.

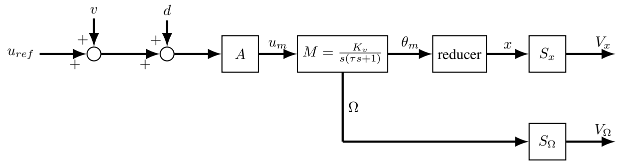

Fig. 4: Plant with position and tachometric sensors

In the following a short technical description of the scheme components is given.

1) Sensors: The position sensor translates a movement ranging from $\{-7.7cm \ldots 7.7cm\}$ into a tension variation of $-15V \ldots 15V$. Thus the transfer function will be in fact a gain with the transfer coefficient $\beta = 1.93\frac{V}{cm}$.

The tachometric generator linked to the engine and will provide a tension proportional with the rotational velocity of the engine. In both cases the sensors are considered to be simple gains that transform their specific physical entry into a tension output.

2) Power amplifier and engine: The power amplifier has an unitary gain in tension. Its role is to give the necessary current intensity, necessary to the input of the engine. Several parameters define the engine, we mention Φ_0, the flux constant considered for equal couple, R, L and J the resistance, inductance and inertial characteristic. Finally, a mechanical viscosity coefficient α is considered.

The transfer function is defined as the rapport between the output angular position θ_m and the input electrical tension u_m. The additional signals of induced current i, λ, the mechanical couple and Ω, the angular velocity will be also used to obtain the transfer function.

The relevant equations are detailed below:

$$\begin{aligned} u_m(t) &= Ri(t) + L\frac{di(t)}{dt} + \Phi_0\Omega(t) \\ \lambda(t) &= J\frac{d\Omega(t)}{dt} + \alpha\Omega(t) \\ \lambda(t) &= \Phi_0 i(t) \end{aligned} \quad (18)$$

Applying the Laplace transformation the transfer function:

$$\frac{\Omega(s)}{U_m(s)} = \frac{\Phi_0/\left(\alpha R + \Phi_0^2\right)}{\frac{LJ}{(\alpha R+\Phi_0^2)}s^2 + \frac{R(J+\alpha L/R)}{(\alpha R+\Phi_0^2)}s + 1} \quad (19)$$

We make the notations $K_v = \frac{\Phi_0}{(\alpha R+\Phi_0^2)}$ and $\tau = \frac{RJ}{(\alpha R+\Phi_0^2)}$ and in collaboration with $\theta(s) = \Omega(s)/s$ and assuming $L/R \approx 0$ the engine transfer function is written as:

$$\frac{\theta(s)}{U_m(s)} = \frac{K_v}{s(1+\tau s)} \quad (20)$$

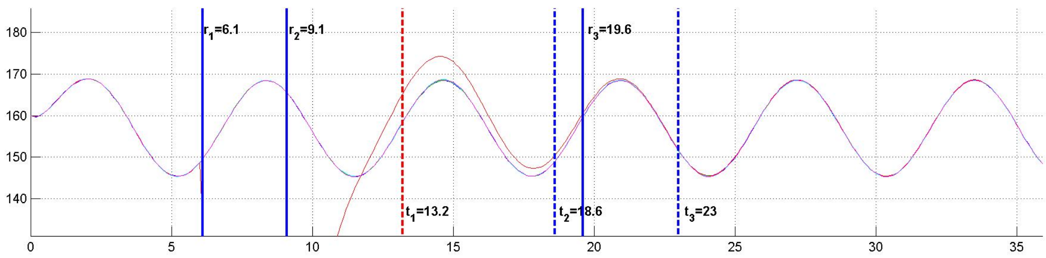

Fig. 3: Example of functioning

The reducer has an exchange rate of $1/N = 1/7$ and an inertia equal with $\frac{J_c}{N^2}$ which is negligible for the given numeric values. The pulley transforms the rotation θ_s into an linear deplacement $x(t)$. The transfer function is then a gain proportional with the wheel radius $\frac{\delta x}{\delta \theta_s} = \rho$ with the numerical value of $\rho = 2.16\frac{cm}{rad}$.

Practically, the system parameters are determined through a least square methodology. The response obtained by exciting the system on a spectrum of frequencies between 3 and 30 Hz is analyzed for its amplitude and phase components in order to obtain the parameters K_v and τ.

The numerical values of the parameters are $K_v = 3.5$ and $\tau = 0.019$ and the command law in the form $u = Kx$ with $K = \begin{bmatrix} 1.18 & 7 \cdot 10^{-3} \end{bmatrix}$ is employed such that the system is stabilised with poles $\begin{bmatrix} 0.5 & 0.9 \end{bmatrix}$.

In order to use the fault tolerance scheme some additional information is required. Firstly, bounds for all the noises affecting the system must be determined. We will consider the noise affecting the plant w and the sensors in both healthy and faulty mode of functioning η_i and η_i^F, respectively. It must be stated that the noises analysed have a gaussian distribution and therefore they can have arbitrarily high values. However, from a practical point of view we chose a set of bounds such that the probability of an actual passage is considered negligible. The numerical values obtained are $\bar{w} = 0.2$, $\eta_1 = 0.1, \eta_1^F = 0.3$ and $\eta_2 = 0.5, \eta_2^F = 1$.

The command input as well as the output received are hardware limited. The values, expressed in voltages must be restricted to the sets

$$\mathcal{U} = \{u: -10V \leq u \leq 10V\} \quad (21)$$
$$\mathcal{Y} = \{y: -9.85V \leq y \leq 9.85V\} \quad (22)$$

for the input command and respectively for the output (this value was obtained by measuring the voltage output for a maximal elongation of the cursor in both directions). These limitations in turn impose limits on the possible values of the reference signal x_{ref} that can be used. In order to respect these limitations the minimal output set O_∞ will be defined [9]:

$$O_\infty(k) = \{x_{ref}(k): u(k+t) \in \mathcal{U}, \, y(k+t) \in \mathcal{Y}, \quad t \geq 0\} \quad (23)$$

Finally, each pair (A, C_i) must be observable, a condition which is not true in the case of the tachometric generator. In consequence, a composite sensor, considered as a sum of the both real sensors will be used. The outputs will in then be determined by $C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}$ and $C_2 = \begin{bmatrix} 1 & 0.026 \end{bmatrix}$.

B. Results

For the above system a hybrid structure was employed in the sense that, the command structure was software implemented and the positioning system was hardware. The two parts were interconnected through a acquisition board.

In Figure 3 an simple example is presented. One can see that, even if, at moment $t = 6s$ a sensor becomes faulty the plant state suffers no visible degradation (the reference is still followed) thanks to the FDI mechanism.

VI. Conclusions

This paper provides an effective method of fault tolerant control of a multisensor scheme. A robust FDI mechanism that implements a set membership approach is presented. Offline computations of invariant sets are performed and, only set membership tests are performed at run time. A detailed example of a positioning system is analysed.

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