A Modified Level Set Model for Mammograhic Masses Segmentation

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Abstract —**Chan-Vese (C-V) model is based on Mumford-Shah and level set methods. We introduce a scheme for segmentation of mammograhic masses, by adding a new regularizing term and morphological top hat transform to C-V model. The accuracy is analyzed quantitatively using area overlap ratio (AOR), tested on 50 masses from Digital Database for Screening Mammography (DDSM). At overlap threshold of 0.5, the proposed scheme and C-V method correctly segment 84% and 66% of the masses, respectively.**

***Keywords* — Geometric Active contour, Level Sets, Mammograms, Segmentation.**

I. Introduction

Mass segmentation is an essential step in computer-aided diagnosis (CAD) for digital mammograms. The task of segmentation of mammographic masses is not trivial, since mass lesions are usually embedded in parenchymal structures of the female breast. Segmentation algorithms for medical images are classified into three categories: threshold based algorithms, pattern recognition based techniques; and deformable based models [1]. Deformable models are classified into: parametric contour models which based on the snake model introduced by Kass et al. [2]; and geometric contour models based on level set method proposed by Osher and Sethian [3]. The Chan-Vese (C-V) model introduced by Chan and Vese [4] is based on Mumford-Shah model [5] and level set method [3]. Zhang et al. [6] introduced the gradient of the image to C-V model to segment interior contours of medical images. Xiao et al. [7] improved the speed of image segmentation of C-V model.

In this work we introduce the morphological transform and a new regularizing term to the C-V model, to segment the mammographic masses of the female breast.

The rest of the paper is organized as follows: section II introduces the C-V model. Section III describes the proposed model. The level sets formulation of the proposed model is introduced in section IV. Results and discussions are presented in Section V. Finally, conclusions and directions for future work are given in section VI.

II. The C-V Model

The geometric Active contour model proposed by Chan and Vese [4], relies on Mumford-Shah model and level set method. The C-V model is not based on an edge-function to stop the evolving contour on the desired object boundary. It is region-based model, which includes analysis of homogeneities inside and outside of the evolving contour.

Let Ω be a bounded open subset of R^2, with its boundary $\partial\Omega$. Let f_0 be a given image, and C be a contour on the image. This model can be expressed mathematically by the following energy function as follows,

$$\begin{aligned} E(c_1, c_2, C) = {} & \mu \cdot Length(C) \\ & + \lambda_1 \int_{inside\ C} (f_0 - c_1)^2 dx\,dy \\ & + \lambda_2 \int_{outside\ C} (f_0 - c_2)^2 dx\,dy \end{aligned} \tag{1}$$

where $\mu \geq 0$, $\lambda_1, \lambda_2 > 0$ are fixed weight parameters, C is the evolving contour and *Length* (C) is a regularizing term that prevents the final contour from converging to a small area due to noise, and c_1 and c_2 are mean values inside and outside of the curve C , respectively.

III. The Proposed Model

Our model is based on adding a new regularizing term to the C-V model, combined with the top hat morphological (THM) transform of the image [8]. Here, we introduce the THM of the image to the C-V model (1) by choosing a kernel function *K* instead of f_0 as follows,

$$\begin{aligned} E(a_1, a_2, C) = {} & \mu \cdot Length(C) \\ & + \lambda_1 \int_{inside\ C} (K - a_1)^2 dx\,dy \\ & + \lambda_2 \int_{outside\ C} (K - a_2)^2 dx\,dy \end{aligned} \tag{2}$$

$$K(x, y) = THM(f_0(x, y)) \tag{3}$$

where a_1, a_2 are the averages of *K* inside and outside C , respectively; and $THM(f_0)$ is a smoother version of f_0, and can be get as follows,

$$THM(f_0) = f_0 + T_{Op} - T_{Cl} \tag{4}$$

where T_{Op} is the difference between the original image and the one processed by the opening operator, T_{Cl} is difference between the original image processed by the

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closing operator and the original image.

The proposed regularizing term is added to C-V model, this new regularizing term RT is defined below:

$$RT = \frac{\tau}{2}\int_{\Omega} |\nabla \phi|^2 dx\,dy \tag{5}$$

where $|\nabla \phi| = \sqrt{\phi_x^2 + \phi_y^2}$, $\phi_x = \frac{\partial \phi}{\partial x}, \phi_y = \frac{\partial \phi}{\partial y}$

The term RT is added to the active contour model in equation (2), to keep the level set function ϕ close to a signed distance function (SDF) in Ω; hence, gives simplicity and efficiency in using signed distance function. By combining (2) and (5) we can get the a new energy function which is expressed as follows,

$$\begin{aligned} E(a_1, a_2, C) = {} & \mu \cdot Length(C) \\ & + \frac{\tau}{2}\int_{\Omega} |\nabla \phi|^2 dx\,dy \\ & + \lambda_1 \int_{inside\ C} (K - a_1)^2 dx\,dy \\ & + \lambda_2 \int_{outside\ C} (K - a_2)^2 dx\,dy \end{aligned} \tag{6}$$

where τ is a weighted parameter, and μ controls the smoothness of the final contour. For detection of all or as many objects as possible and of any size, one should choose a small μ, but for large objects and with smoother contour, one should set a large μ.

IV. FORMULATION OF THE PROPOSED MODEL VIA LEVEL SETS

Level set theory [3], in which the two-dimensional evolving contour C is represented implicitly as the zero level set of a three-dimensional Lipschitz function $\phi : \Omega \to R$, which is expressed as follows,

$$\begin{aligned} inside\ C &= \{(x,y) \in \Omega : \phi(x,y) > 0\} \\ at\ C &= \{(x,y) \in \Omega : \phi(x,y) = 0\} \\ outside\ C &= \{(x,y) \in \Omega : \phi(x,y) < 0\} \end{aligned} \tag{7}$$

These equations evolve the contour by updating the level set function $\phi(x, y)$ at fixed coordinates through iterations instead of tracking the contour itself. The level set function $\phi(x, y)$ is usually chosen as the distance to the initial contour $\phi_0(x, y)$, and is positive if (x, y) is inside C, negative if (x, y) is outside C.

By replacing C by $\phi(x, y)$ and introducing the Heaviside function H_ε and the corresponding Dirac measure δ_ε as follows,

$$H_\varepsilon(z) = \frac{1}{2}\left(1 + \frac{2}{\pi}\arctan(\frac{z}{\varepsilon})\right), \quad \delta_\varepsilon(z) = \frac{d}{dz}H_\varepsilon(z) \tag{8}$$

Hence, as in the C-V model [4], equation (6) can be expressed as follows,

$$\begin{aligned} E_\varepsilon(a_1, a_2, \phi) = {} & \mu \int_{\Omega} \delta_\varepsilon(\phi(x,y)) |\nabla \phi(x,y)|\, dxdy \\ & + \frac{\tau}{2}\int_{\Omega} |\nabla \phi(x,y)|^2\, dxdy \\ & + \lambda_1 \int_{\Omega} (K - a_1)^2 H_\varepsilon(\phi(x,y))dxdy \\ & + \lambda_2 \int_{\Omega} (K - a_2)^2 (1 - H_\varepsilon(\phi(x,y)))dxdy \end{aligned} \tag{9}$$

where a_1 and a_2 are calculated as follows,

$$a_1 = \frac{\int_{\Omega} K(x) H_\varepsilon(\phi(x))dx}{\int_{\Omega} H_\varepsilon(\phi(x))dx}, \quad a_2 = \frac{\int_{\Omega} K(x)(1 - H_\varepsilon(\phi(x)))dx}{\int_{\Omega} (1 - H_\varepsilon(\phi(x)))dx} \tag{10}$$

Euler-Lagrange equation for $\phi(x, y)$ is derived by fixing a_1, a_2, and minimizing E_ε in terms of $\phi(x, y)$; and the evolution of $\phi(x, y)$ is expressed as follows:

$$\begin{aligned} 0 = {} & \delta_\varepsilon(\phi)\left[\mu.div(\frac{\nabla \phi}{|\nabla \phi|}) - \lambda_1 (K - a_1)^2 + \lambda_2 (K - a_2)^2\right] \\ & + \tau.div(\nabla \phi) \end{aligned} \tag{11}$$

Equation (11) can be solved by discretizing ϕ using the gradient descent method; by letting ϕ be a function of iteration $t \geq 0$ and replacing the zero on the left-hand side by the time derivative of ϕ. Thus, we obtain a partial differential equation (PDE) in $\phi(x, y, t)$, with initial contour $\phi(x, y, 0) = \phi_0(x, y)$, as follows,

$$\begin{aligned} \frac{\partial \phi}{\partial t} = {} & \delta_\varepsilon(\phi)\left[\mu.\kappa - \lambda_1 (K - a_1)^2 + \lambda_2 (K - a_2)^2\right] \\ & + \tau.\Delta\phi \end{aligned} \tag{12}$$

where $\Delta\phi = \phi_{xx} + \phi_{yy}$ and κ are the Laplacian and the curvature (or divergence) of the evolving contour ϕ, respectively. For the discretization of the divergence and Laplacian operators and the iterative algorithm are from [9]. The initial contour $\phi_0(x, y)$, we used here is a circle defined as follows,

$$\phi_0(x, y) = -\sqrt{(x - x_0)^2 + (y - y_0)^2} + r \tag{13}$$

where (x_0, y_0) and r are the center, and the radius of the circle, respectively.

A. *Summary of the Proposed Scheme:*

The proposed level set scheme for segmentation of mammographic mass images can be summarized as follows:

- Process the original image f_0 using equation (4).

- Initialize the implicit active contour (zero level set) function ϕ_0 using equation (13).
- Compute a_1 and a_2 using equation (10).
- Segment mammographic masses boundary using the evolving contour by solving the iterative PDE (12).

V. Experimental Results and Discussions

In this study, we developed automatic segmentation scheme for mammographic mass segmentation in the female breast. The proposed method is a geometric active contour model (or level set model), which adds a new regularizing term and top hat transform of the image to C-V model. The data set of mammograms used in this study is selected from publicly available database, the Digital Database for Screening Mammography (DDSM) established by a research group at the university of south Florida [10].

To evaluate the performance of the proposed approach, we made some experiments using MATLAB™ software version 7.4 in a desktop computer Pentium IV with dual CPU 1.6 GHz and 1 GB RAM. The proposed approach was applied on 50 mammographic mass images selected from the DDSM database.

We kept $\lambda_1 = \lambda_2 = 1$ in equation (12), since the contribution of the homogeneities inside and outside the evolving contour should be equally considered. Other parameters were chosen as follows: $\varepsilon = 1$ and $\Delta t = 0.1$, where ε influences the Heaviside function and Δt controls how quickly the level set function changes.

The proposed method doesn't require any user input, we fix the maximum number of iterations to 40, r to 15 pixels, (x_0, y_0) to (62.5,62.5), τ to 9, and μ to 0.02*125*125. For equation (4) the structuring element used here in our experiments is a disc with radius 20.

The performance analysis of the proposed segmentation scheme on the test images is assessed by a quantitative measure: area overlap ratio (AOR) [11], between computer segmentation and manual segmentation which is defined as follows,

$$AOR = \frac{R \cap S}{R \cup S} \tag{14}$$

Where R is the area of the segmented contour drawn manually from the ground truth of the masses in DDSM database, and S is the area of the segmented contour by the proposed method (computer segmentation). For the entire test data AOR is obtained and fraction correctly segmented (FCS) masses is calculated at each AOR threshold by counting number of masses with AOR greater than that threshold.

The segmentation results of the proposed model and C-V model were compared empirically. Figure 1 shows superior AOR values of the proposed method against C-V model for 50 masses. At the overlap threshold of 0.5, for mass lesions, 84 % of the images are correctly segmented with the proposed segmentation method, while 66 % of the images are correctly segmented by the C-V algorithm. Figure 2 shows several examples of segmented masses using two segmentation methods. The result of the proposed method visually demonstrates a better agreement with the DDSM ground truth contour of the mass drawn by expert radiologist, as shown in figure 2.

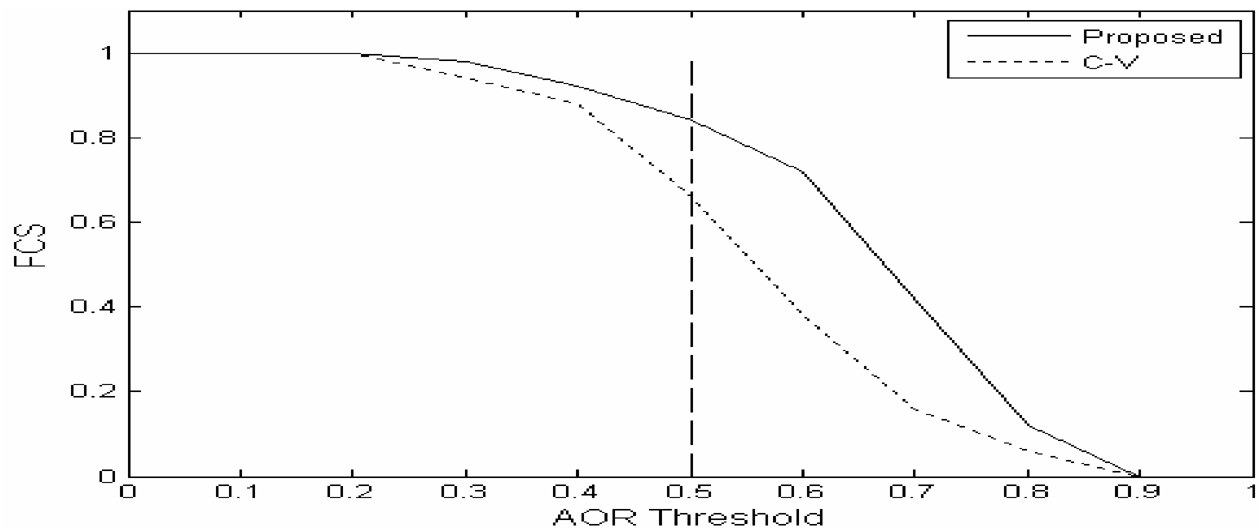
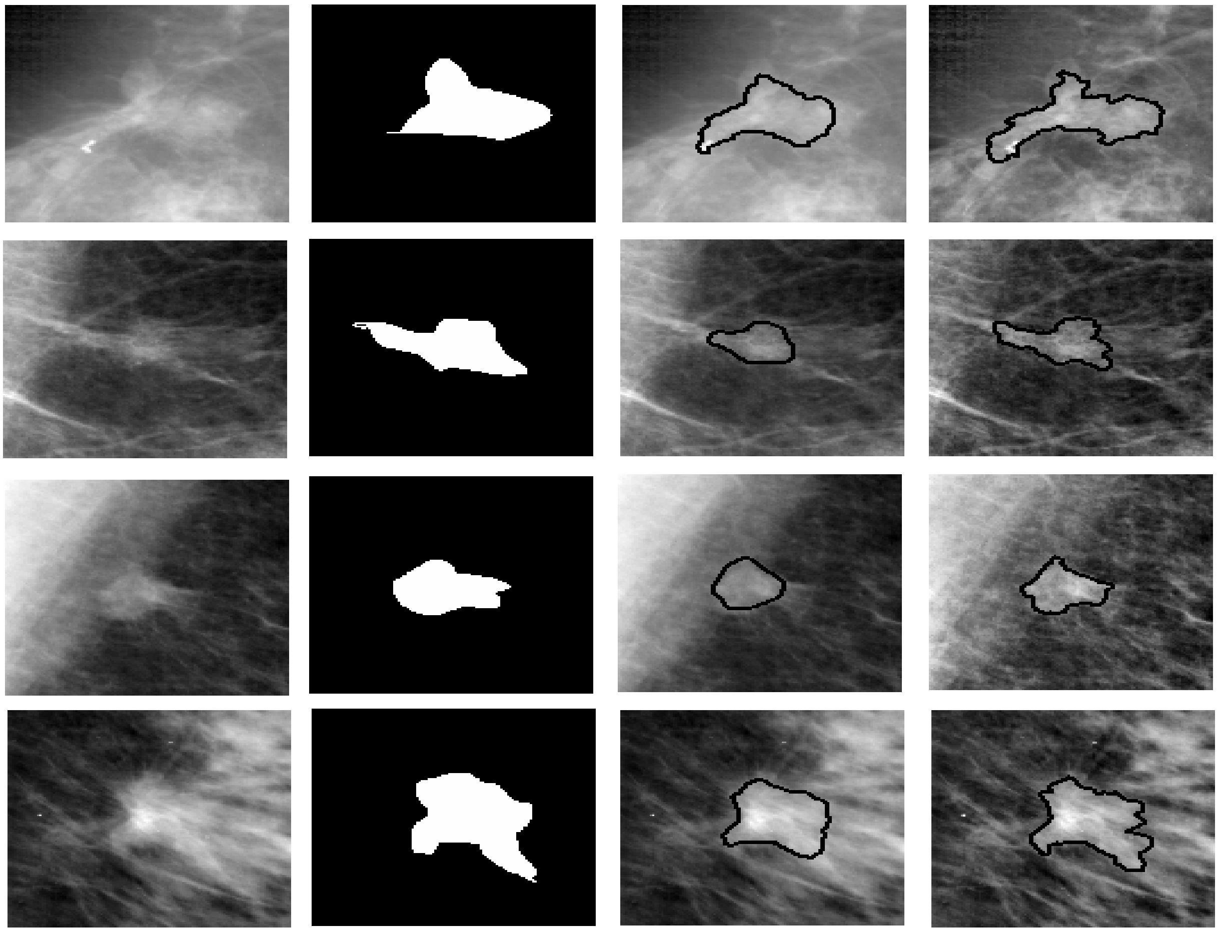


Fig. 1: Fraction correctly segmented (FCS) masses at different Area overlap ratio (AOR) threshold levels



a b c d

Fig. 2: Segmentation results for mass lesions (a) the original image (b) the manual contour marked by expert radiologist from DDSM database, (c) C-V segmentation method, and (d) the proposed segmentation method.

VI. Conclusion

In this paper, we introduce a mass segmentation method based on a modified level set model. It is based on adding a new regularizing term and top hat morphology transform to C-V model. Experimental results for segmentation of mammographic masses via the proposed method yield significant improvement when it is compared to C-V algorithm. The next step is to extend our work to large data set and improve this method by making the evolution process of the contour not depending on fixed number of iterations.

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