

# Adaptive Channel Estimation based on Soft Information Processing in Broadband Spatial Multiplexing Receivers

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**Abstract**—In this paper we present a novel approach in Multiple-Input Multiple Output (MIMO) Orthogonal Frequency Division Multiplexing (OFDM) channel estimation technique based on a Decision Directed Recursive Least Squares (RLS) algorithm in which no pilot symbols needs to be integrated in the data after a short initial preamble. The novelty and key concept of the proposed technique is the block-wise causal and anti-causal RLS processing that yields two independent processing of RLS along with the associated decisions. Due to the usage of low density parity check (LDPC) channel code the receiver operates with soft information, which enables us to introduce a new modification of the Turbo principle as well as a simple information combining approach based on approximated a-posteriori log-likelihood ratios (LLRs). Although the computational complexity is increased by both of our approaches, the latter is relatively less complex than the earlier. Simulation results show that these implementations outperform the simple RLS-DDCE algorithm and yield lower bit error rates (BER) and more accurate channel estimates.

## I. INTRODUCTION

A widespread modulation technique used in today's communication systems is orthogonal frequency-division multiplexing (OFDM), which combines high spectral efficiency, robustness against inter-symbol interference and an easy implementation using the fast Fourier transform (FFT). Combining the OFDM system with a multiple-input-multiple-output (MIMO) system, a MIMO-OFDM system is created, which results in a higher spectral efficiency and link reliability [1].

Especially under bad transmission conditions with small signal-to-noise ratios (SNR) or high mobility, there are broad possibilities to improve the performance of MIMO-OFDM systems. High mobility involves a highly time-variant channel, which causes the spectral efficiency to decrease. A solution to counteract this degradation is the enhancement of detailed knowledge of the channel state information (CSI).

Receiver designs for MIMO-OFDM which make acceptable use of diversities are rare. There are few researches focusing on iterative receiver architecture [2], [3], which exploit the Turbo principle with its iterative decoding structure. Even though they result in higher computational complexity these receivers seem to be promising in relation to BER performance. Since the LDPC codes possess similar performance when compared

to Turbo Codes, they are implemented in MIMO-OFDM systems as well [4].

In this paper we propose a novel turbo processing based on the LDPC codes, where the information gain is obtained from a causal RLS-DDCE processing and an independent anti-causal processing. These two possible strategies are presented here. Simple information combined by summing up of a-posteriori information and Turbo processing by exchange of extrinsic information between forward RLS-DDCE process and backward fork.

The rest of the paper is organized as follows. The underlying system model and structure are presented in section II, followed by the description of the RLS-DDCE algorithm in section III. Our novel approach with detailed information about the modified Turbo principle and summation of a-posteriori LLRs is presented in section IV, respectively in subsection IV-A and IV-B. The paper is concluded by illustrating our simulation results in section V and a conclusion in section VI.

## II. SYSTEM MODEL AND STRUCTURE

The vector of received values  $\mathbf{r}$  at the time sample  $m$  of a MIMO system is the superposition of  $L \cdot n_T$  previously sent samples and the current  $n_T$  samples, where  $L+1$  is the length of the sampled channel impulse response and  $n_T$  is the number of transmit antennas. It is given by

$$\mathbf{r}[m] = \sum_{l=0}^L \mathbf{h}[l, m] \cdot \mathbf{s}[m-l] + \mathbf{w}[m], \quad (1)$$

where  $\mathbf{s}[m]$  denotes the current vector of symbols of each of the transmit antenna,  $\mathbf{w}$  is an identically, independently distributed (iid) additive white Gaussian noise term and  $\mathbf{h}[l, m]$  is the MIMO channel matrix in delay and time domain, indexed with  $l$  respectively  $m$ . The past sent samples are denoted by  $\mathbf{s}[m-l]$ , for  $l \neq 0, l \leq L$ . For simulations the data symbols of the  $K$  subcarriers are modulated by an inverse Fast Fourier Transform (IFFT). In simulations every value corresponding to a transmit antenna of the resulting vectors is transmitted using the formula above.

In frequency domain the system model in equation (1) can be described as

$$\mathbf{r}[n, k] = \mathbf{H}[n, k] \cdot \mathbf{s}[n, k] + \mathbf{w}[n, k], \quad (2)$$

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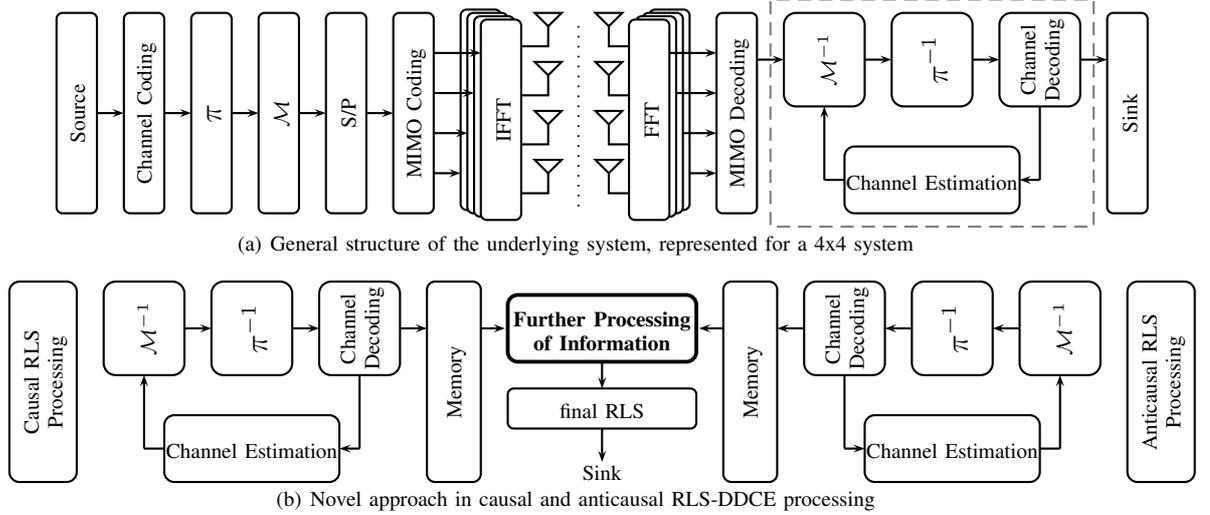


Fig. 1. Illustration of the system structure in Figure 1(a) and our proposed improvement, implemented on the receiver side of the system, in Fig. 1(b)

where  $n$  denotes the time index of an OFDM symbol and  $k$  its subcarrier index. The vector  $\mathbf{r}[n, k]$  is of dimension  $n_R \times 1$ ,  $\mathbf{s}[n, k]$  and  $\mathbf{w}[n, k]$  of  $n_T \times 1$  and the matrix  $\mathbf{H}[n, k]$  of  $n_R \times n_T$ , at which  $n_R$  is the number of transmit antennas. In simulations the MIMO channel coefficients  $H_{r,t}[n, k]$ ,  $r = 1, \dots, n_R$ ,  $t = 1, \dots, n_T$  are modeled using the 3GPP spatial model which is developed to evaluate receiver algorithms in MIMO scenarios [5].

An overview of the implemented general system structure can be seen in Fig. 1(a). Fig. 1(b) shows the structure of our novel approach to the channel estimation and is explained in section IV.

### III. RLS-DDCE

The RLS algorithm as described in [6] is suitable for tracking a communication channel as it computes an estimate of the current channel matrix  $\hat{\mathbf{H}}[n, k]$  upon arrival of new received data  $\mathbf{r}[n, k]$  and converges within just a few OFDM symbols.

The introduced forgetting factor  $\xi$  associates an exponential weighting of the past transmitted signals onto the current channel factor. Therefore it can be used to adapt to the time-variant channel conditions.

For calculating the channel transfer function the autocorrelation matrix (3) of the transmitted signals and the cross-correlation matrix (4) of the transmitted and received signals are needed.

$$\Phi[n, k] = \xi \cdot \Phi[n-1, k] + \mathbf{s}[n, k] \cdot \mathbf{s}^H[n, k] \quad (3)$$

$$\theta[n, k] = \xi \cdot \theta[n-1, k] + \mathbf{s}[n, k] \cdot \mathbf{r}^H[n, k] \quad (4)$$

To obtain an estimate of the channel matrix the normal equation can be used as follows:

$$\hat{\mathbf{H}}[n, k] = (\Phi^{-1}[n, k] \cdot \theta[n, k])^H. \quad (5)$$

Summation of rank-1 matrices in (3) and (4) is avoided by starting each transmission frame with a training sequence of known pilot symbols, as the matrices have full condition after a few summations. The transmitted symbols are known in the receiver and the channel transfer function estimates  $\hat{\mathbf{H}}[n, k]$  can instantly be calculated. The symbols following the pilot symbols have to be decided in the receiver, for that reason the received symbols are equalized, decoded and detected before calculation of the channel factors.

### IV. FORWARD AND BACKWARD RLS PROCESSING

The RLS-DDCE algorithm provides the receiver with information on the channel and also results in the transmitted signals. Our novel proposal for this is to perform the RLS algorithm twice, in causal and anti-causal direction, which uses block-wise processing of received data, as shown in Fig. 2. Fig. 1(b) illustrates our proposed approach. On the left side the causal RLS processing is shown, which yields the a-posteriori LLRs  $L_1(u_h|\mathbf{y})$  out of the received symbols  $\mathbf{r}[n, k]$  by soft demodulation and decoding, depicted by  $\mathcal{C}^{-1}$  in equation (6). The right part of the figure processes the received data anti-causal wise, equation (7), where the length of the pilot symbols is denoted by  $N_P$  and the data length by  $N_D$ . This routine can be described as:

$$L_1(u_h|\mathbf{y}) = \mathcal{C}^{-1}(\mathcal{M}^{-1}\{\mathbf{H}^\dagger[n, k]\mathbf{r}[n, k]\}), \quad (6)$$

$$L_2(u_h|\mathbf{y}) = \mathcal{C}^{-1}(\mathcal{M}^{-1}\{\mathbf{H}^\dagger[\nu, k]\mathbf{r}[\nu, k]\}), \quad (7)$$

$$\nu = (N_P + N_D) - n,$$

where  $\mathbf{H}^\dagger$  denotes the pseudo-inverse of the channel matrix and  $\nu$  the anti-causal time index. The underlying frame structure only provides a training sequence at the beginning of each frame, and therefore the training sequence of the subsequent transmission frame can be used for the anti-causal RLS processing. Exploiting the incremental overhead

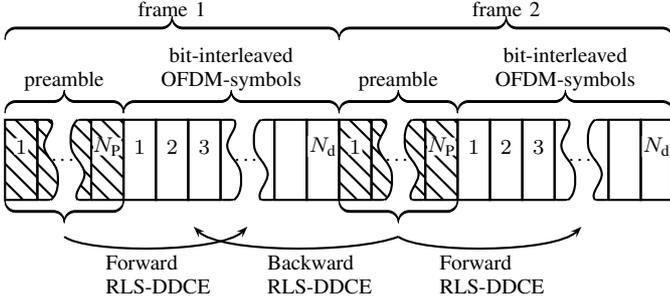


Fig. 2. Frame structure for proposed MIMO-OFDM RLS-DDCE Forward and Backward Filtering with re-use of next and previous frame preambles.

twice comes at no additional overhead cost, but results in an additional channel information gain. Fig. 2 gives an idea of the double usage of the pilot symbols.

### A. Modified Turbo Principle

For our purpose the original Turbo principle of [7] and [8], is changed, as presented in Fig. 3. We totally ignore the encoding part of the original Turbo Coding and solely perform the normal LDPC coding as presented in section II, which will lead to the combination of LDPC and Turbo decoding in the receiver.

On the receiver side we retain the Turbo decoding layout, though we change the inputs to the component decoders. As the RLS algorithm is processed in causal and anti-causal manner the soft information of the received bits  $y_h$  is available twice,  $y_{h,1}$  and  $y_{h,2}$ . The availability of two different inputs, which are supposed to be the same under perfect conditions, replaces the usage of two different codes. The extrinsic information is created and exchanged in the same way as in the original Turbo principle:

$$L_e(u_h) = L(u_h|\mathbf{y}) - L(u_h) - L_c \cdot y_h, \quad (8)$$

where  $L_c$  denotes the channel reliability.

Our proposed Turbo decoding is performed twice, starting with the soft information from the causal RLS first, followed by the anti-causal information with the adequate extrinsic information (9) and vice versa (10).

$$L_1(u_h|\mathbf{y}) \Rightarrow L_{1,e}(u_h) = L_2(u_h) \Rightarrow L_2(u_h|\mathbf{y}) \quad (9)$$

$$L_2(u_h|\mathbf{y}) \Rightarrow L_{2,e}(u_h) = L_1(u_h) \Rightarrow L_1(u_h|\mathbf{y}) \quad (10)$$

In our proposal we use the break criterion provided by [9].

An additional break criterion in order to avoid unnecessary iterations can be derived from the PCS. For all codewords of either the causal or anti-causal RLS are zero, which is mostly the case for high SNR, the iteration of the Turbo decoding will not even start. The comparably large codeword distance of LDPC codes [10] can be used to evaluate the performance of certain iteration steps. A PCS of zero is likely to be equivalent to an error free codeword. And in addition, simulations have

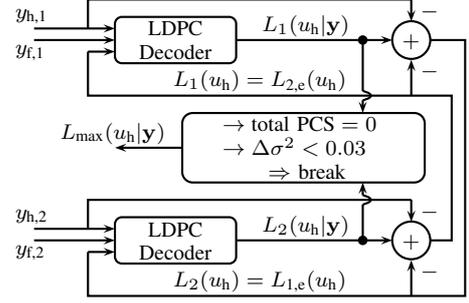


Fig. 3. Modified Turbo Principle

shown that a small total PCS over all codewords is connected to a lower BER. The total PCS is then used to decide on the output of the two different iteration directions. The lower total PCS of the second component decoder determines the a-posteriori LLRs  $L_{\max}(u_h|\mathbf{y})$  that are to be further processed. The two break criteria are also shown in Fig. 3.

### B. Summation of a-posteriori LLRs

Our second approach to improve the channel estimation is to simply sum up the a-posteriori LLRs from the forward and backward RLS processing:

$$L_{\max}(u_h|\mathbf{y}) = L_1(u_h|\mathbf{y}) + L_2(u_h|\mathbf{y}). \quad (11)$$

This, in comparison to the modified Turbo principle, has less computational complexity and still corrects a large amount of errors. The reliable codewords to conduct the final RLS processing are determined by comparing the hard decided bits, based on the a-posteriori LLRs after the summation, with the hard decided bits of the a-posteriori LLRs before the summation. A codeword is considered to be reliable when all a-posteriori LLRs of a certain codeword do not change due to the summation.

## V. SIMULATION RESULTS

TABLE I  
MIMO-OFDM SYSTEM PARAMETERS

Parameter	Value
carrier frequency	2.412 GHz
channel bandwidth	20 MHz
Number of subcarriers $K$	128
Pilot rate	5.6%
channel model	3GPP Spatial Channel Model
LDPC design code rate	1/2

The simulation was performed on a  $4 \times 4$  MIMO-OFDM system with the simulation parameter given in Table I. For the modulation, a 4-QAM was taken so that an OFDM symbol consisted of 1024 bits. The calculated frame duration based on the parameters resulted in 3.86 ms. The forward, backward and final RLS processing used the simple zero forcing equalizer due to computational complexity and retention of soft information. The forgetting factor  $\xi$  was chosen according to [6] with a value of 0.7, so that the algorithm worked fine

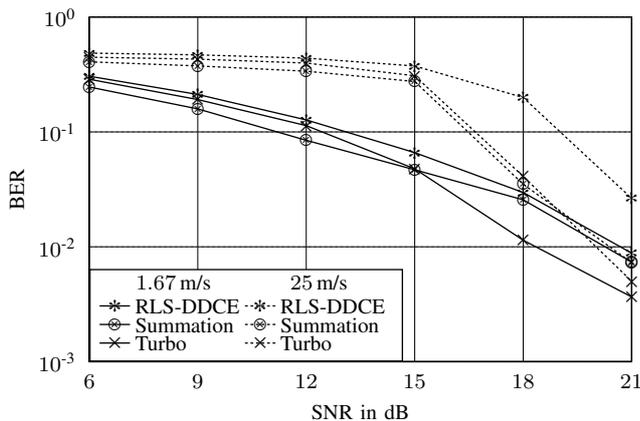


Fig. 4. BER for different SNR at velocities of 1.67 m/s and 25 m/s

over a large range of velocities. Fig. 4 shows the comparison of the implemented algorithms. Over the entire SNR range the simple RLS-DDCE performed worst. For smaller SNR, up to about 15 resp. 19 dB, the summation of a-posteriori LLRs dominated the BER performance. The Turbo principle was slightly worse, but increased in performance for higher SNR values. For small SNR values the receiver did not yield correct codewords, so the performance increase in comparison to the simple RLS-DDCE algorithm was solely due to the extended soft information evaluation of the final RLS processing. In addition the summation used the added a-posteriori LLRs for the BER evaluation in low SNR regions, which explains the better performance for small SNR values. For the upper SNR range the Turbo principle worked better due to the larger amount of correct codewords, which resulted in a smaller BER.

Fig. 5 shows the iterative behavior of the implemented Turbo principle for a SNR of 18 dB and a velocity of 8.33 m/s. The figure depicts how the number of codewords with a total PCS of zero increased with increasing number of iterations, for the forward and backward iteration. Along with it the number of wrong codewords, codewords with PCS of zero and biterrors, also rose. This was due to the LDPC decoder, which ran into wrong codewords due to the exchange of extrinsic information. The curve's slope for the total PCS was flattening with increasing iterations, so that the variance break criterion became active at one point and stopped the iteration. In case the final RLS processing is able to make better decisions with more reliable channel information, then the number of iterations should not be too large in order to avoid incorrect codewords. In addition Fig. 5 presents the difference between the causal and anticausal iteration direction, as can be seen at the starting values of the curves.

## VI. CONCLUSION

In this paper we have presented a novel approach to the channel estimation process for challenging time-variant channels and the performance has been evaluated for different velocities. The modified Turbo principle, based on different input data for the component decoder, shows increased performance over the entire velocity range for larger SNR values, especially

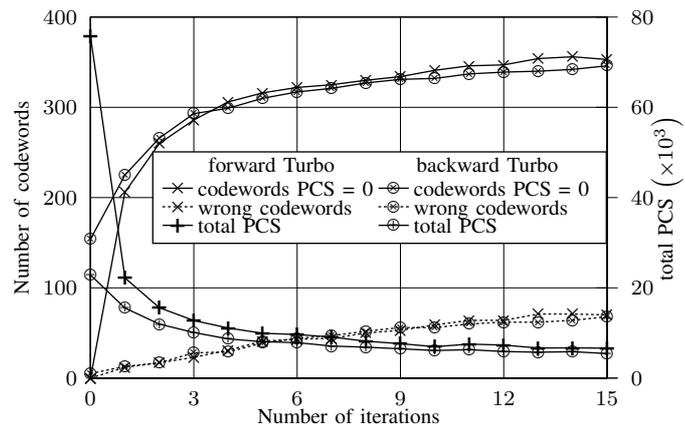


Fig. 5. Iterative behavior of the Turbo principle showing the number of correct codewords, wrong codewords and total PCS for the forward and backward Turbo iteration, at a SNR of 18 dB and 8.33 m/s

at the higher velocity range the performance compared to the simple RLS-DDCE is superior. At lower SNR values the performance is still better than the simple RLS-DDCE, though the applied zero forcing equalizer prohibits better performance. The summation of a-posteriori LLRs in contrast performs better for smaller SNR values as the summation corrects a certain amount of wrong decided symbols. The BER performance increases due to better channel estimation for the proposed approaches.

## REFERENCES

- [1] H. Blcskei, D. Gesbert, and A. Paulraj, "On the capacity of OFDM-based spatial multiplexing systems," *IEEE Transactions on Communications*, vol. 50, no. 2, pp. 225–234, Feb. 2002.
- [2] J. Akhtman and L. Hanzo, "Iterative receiver architectures for MIMO-OFDM," in *Wireless Communications and Networking Conference, 2007.WCNC 2007. IEEE*, Mar 2007, pp. 825–829.
- [3] H. Liu, X. Wang, and Z. Xiong, "Iterative receivers for OFDM coded broadband MIMO fading channels," *IEEE Workshop on Statistical Signal Processing*, pp. 355–358, Oct. 2003.
- [4] B. Lu, G. Yue, and X. Wang, "Performance analysis and design optimization of LDPC-coded MIMO OFDM systems," *IEEE Transactions on Signal Processing*, vol. 52, pp. 348–361, Feb. 2004.
- [5] *Spatial channel model for Multiple Input Multiple Output (MIMO) simulations*, 3rd Generation Partnership Project (3GPP), Technical Specification Group Radio Access Network Technical Report 25.996, Rev. Release 8.0.0, 2008.
- [6] J. Akhtman and L. Hanzo, "Advanced channel estimation for MIMO-OFDM in realistic channel conditions," *IEEE International Conference on Communications, 2007. ICC '07*, pp. 2528–2533, Jun. 2007.
- [7] C. Berrou, A. Glavieux, and P. Thitimajshima, "Near Shannon limit error-correcting coding and decoding: Turbo-codes (1)," *IEEE International Conference on Communications*, vol. 2, pp. 1064–1070, May 1993.
- [8] C. Berrou and A. Glavieux, "Near optimum error correcting coding and decoding: Turbo-codes," *IEEE Transactions on Communications*, vol. 44, pp. 1261–1271, Oct. 1996.
- [9] P. Robertson, "Illuminating the structure of code and decoder of parallelconcatenated recursive systematic (turbo) codes," *IEEE Global Telecommunications Conference*, vol. 3, pp. 1298–1303, Nov. 1994.
- [10] D. MacKay, "Good error-correcting codes based on very sparse matrices," *IEEE Transactions on Information Theory*, vol. 45, no. 2, pp. 399–431, Mar 1999.