A Survey of the Correlation Properties of the Generalized Barker Codes

Valentin A. Mutkov, Nikolay R. Nikolov, Rumen G. Tsakov, and Liliya A. Staneva

Abstract — **The Barker codes (signals) were invented in the beginning of the 1950s of the twentieth century. Due to their positive auto-correlation properties the classical Barker signals have been applied in large number of communication devices.**

Unfortunately, the classical Barker signals do not exist for lengths greater than 13 and can not be used as members of families with small cross-correlation. These drawbacks can be avoided by usage of complex *m*-phase shift keying (*m*>2), which leads to the so-named generalized Barker signals. With regard, the correlation properties of generalized Barker signals are studied in the paper. It is shown that there exist families of generalized Barker signals with small cross-correlation among all members of a family.

The analyzed in the paper families of generalized Barker signals could be successfully applied in the sonars of miniature unmanned underwater vehicles.

Keywords — **Algorithms, correlation properties, generalized Barker codes.**

I. Introduction

AT present the miniature radar and sonar devices are used in many areas and technical applications such as: surveillance and control systems, autopilots of the cars, small scientific and spy vehicles, airplanes and so on. In order to perform successfully their function, the signals, exploited in the miniature radars and sonars, must satisfy strong conditions, concerning their correlation properties [1]. First of all, the side lobes of the auto-correlation functions (ACFs) of the signals should be as small as possible, because they determine the dynamic diapason of the radar and sonar images and the possibility to discover small objects. Second, the cross-correlation functions (CCFs) of all pairs of signals, transmitted by miniature radars and sonars of a system, should be close to zero. This condition guarantees the simultaneous work of all devices of the system with an admissible level of the mutual interferences.

Often the above conditions are combined in one saying simply that the family of the signals, used by the radars and sonars of a system, must have optimal correlation properties.

V. A. Mutkov is with the University of Ruse “Angel Kanchev”, Ruse, Bulgaria, (e-mail: vmutkov@ecs.ru.acad.bg).

N. R. Nikolov is with the University of Shumen “Bishop Konstantin Preslavsky”, Shumen, Bulgaria, (e-mail: niki2_1974@abv.bg).

R. G. Tsakov is with the University of Shumen “Bishop Konstantin Preslavsky”, Shumen, Bulgaria, (e-mail: lz2gx@yahoo.com).

L. A. Staneva is with the University of Burgas “Professor Asen Zlatarov”, Burgas, Bulgaria, (e-mail: anest_bg@bitex.bg).

Due to the importance of the signals with optimal correlation properties they have been extensively studied since the 1950s of the twentieth century [1]-[7]. Despite of the taken efforts, a few families with these abilities are known at present [1], [8]-[13]. With regard the correlation properties of the so-named generalized Barker signals are studied in the paper. The interest to this problem follows from the fact that the generalized Barker signals can be easily formed by miniature hardware with very small consumption.

The paper is organized as follows. First, the basics of the Barker codes (signals) are recalled. After that, the correlation properties of the generalized Barker signals are explored in more details. At the end, the areas of application of the results, obtained in the paper, are presented.

II. Basics of the Barker Codes

The Barker codes (signals) were invented in the beginning of the 1950s of the twentieth century [2]. They are complex radio- or audio-signals formed by phase manipulation (PM) of a train of so-named elementary pulses (or chips) and can be mathematically described as follows:

$$v(t) = \sum_{j=0}^{n-1} U_m . u_0(t - j.\tau_0).\cos(\omega_0 . t + \theta_j) \qquad (1)$$

Here:

- U_m is the amplitude of the elementary pulses (chips);
- $\omega_0 = 2\pi f_0$; f_0 is the carrier frequency;
- $u_0(t) = \begin{cases} 1, & 0 \le t \le \tau_0, \\ 0, & t < 0 \cup t > \tau_0, \end{cases}$;
- τ_0 is the duration of the elementary pulses;
- $\theta_j \in \{(2\pi l)/m;\ l = 0,1,\ldots,m-1\}$ are the initial phases of the elementary pulses, providing the thumb-tack shape of the ACF of the signal;
- n is the length of the train of elementary pulses, forming the Barker code (signal).

Often a PM signal can be studied only by means of its complex amplitude:

$$V(t) = \sum_{j=0}^{n-1} U_m .\zeta(j).u_0(t - j.\tau_0), \qquad (2)$$

where $\{\zeta(j)\}_{j=0}^{n-1}$ is the set of complex amplitudes of the elementary pulses. The elements of the set are the m-th roots of the unity:

$$\zeta(j) \in \{\exp(2\pi i l/m); l = 0,1,\ldots,m-1\},\ i = \sqrt{-1}\ . \qquad (3)$$

For simplicity (but without loss of generality) in the rest part of the paper it will be substituted in (1) and (2):

$$U_m = 1[V] . \quad (4)$$

As a result of (4), the energy of the signals, studied in the paper, will be:

$$E = \frac{U_m}{2} n.\tau_0 [W] = \frac{n}{2}.\tau_0 [W] . \quad (5)$$

The classical Barker signals exploit only binary phase manipulation (i.e. binary shift keying – 2-PSK) [2]. This means that $m = 2$ in (1) and (3).

Due to the thumb-tack shape of their ACF and the simplicity of generation, the classical Barker signals have been applied in large number of communication devices. Unfortunately [3], the classical Barker signals do not exist for lengths *n* greater than 13 and can not form families with large size and small cross-correlation among all pairs of its members. These drawbacks can be avoided by usage of complex *m*-phase shift keying (i.e. *m*>2 in (1) and (3)), which leads to the so-named generalized Barker signals.

Many authors have studied the generalized Barker signals [3], [6]-[13]. As a result up to day generalized Barker signals with lengths $1 \le n \le 36$ are known [11], [12]. The natural question which arises is "for given *n* and *m* is it possible to form families of generalized Barker signals, possessing optimal correlation properties?". This problem is studied in more details in the next part of the paper.

III. The Cross-Correlation Properties of the Generalized Barker Codes

In order to study the cross-correlation properties of the generalized Barker signals a software module was created by means of the Communication Toolbox of the MATLAB. This approach is influenced by the great abilities and functional flexibility of the MATLAB, demonstrated by many authors, working in the area of the communications (for example [13], [14]).

The module uses a data base of all known at present generalized Barker codes (GBC) [3], [6], [7], [10]-[12]. Here it ought to emphasize that the classical Barker codes are viewed as a particular case of the GBCs, because *m*-PSK signal constellation is a subgroup of every *m.k*-PSK signal constellation. Due to this reason the classical Barker codes are included in the database.

After choosing the values of the code length *n* and the type *m* of the PSK, the data base is searched in order to find all generalized Barker codes with desired parameters. This way created family of signals is separated in all possible pairs. After that the CCFs of the pairs are calculated. At the end, it is analyzed which signals form pairs with maximal level of the CCF exceeding the threshold $\sqrt{n}$. Some of these signals are excluded from the initial family in order to obtain a new family with desired cross-correlation properties and maximal possible size.

The choice of the value $\sqrt{n}$ as an acceptable maximum level of the CCF of the pairs of the GBCs is determined by the following reasons. First, the minimal bounds of the maximal level of the CCFs of a family of signals were found by L. R. Welch [16]. After that these results were specified for a single pair of GBC by N. Zhang and S. W. Golomb [17]. Second, the general Welch's bound can be evaluated to the form:

$$L_{\min\max} \ge \sqrt{\frac{N-1}{N.n-1}}\ [V] = \sqrt{\frac{1-(1/N)}{n-(1/N)}}\ [V] , \quad (6)$$

where $L_{\min\max}$ is the minimal maximum of the level of the CCFs and *N* is the size of the family of signals (i.e. the number of the signals which the family comprises). The bound (6) is obtained for the signals with unit energy, i.e. $E = (1/2).\tau_0 [W]$. Now, accounting that the energy of the studied in the paper signals actually is $E = (n/2).\tau_0 [W]$, the formula (6) can be reduced to:

$$L_{\min\max} \ge n.\sqrt{\frac{1-(1/N)}{n-(1/N)}}\ [V] \approx \sqrt{n}\,[V] , \quad (7)$$

for families with large size *N*.

The results of the survey of the cross-correlation properties of known at present GBCs, given in [3], [6], [7], [10]-[12], will be illustrated by following example.

As known [3], for $n = 11$ and $m = 6$ six GBCs, which can not be obtained one from other by so-named trivial transformations, exist. With regard to (4), these sequences can be described by means of the sets of their complex amplitudes as follows:

$$\begin{aligned}
&\{\zeta_1(j)\}_{j=0}^{10} = \\
&= \{\omega^0, \omega^0, \omega^0, \omega^0, \omega^2, \omega^2, \omega^5, \omega^0, \omega^3, \omega^0, \omega^4\}; \\
&\{\zeta_2(j)\}_{j=0}^{10} = \\
&= \{\omega^0, \omega^0, \omega^0, \omega^2, \omega^3, \omega^0, \omega^1, \omega^5, \omega^3, \omega^0, \omega^4\}; \\
&\{\zeta_3(j)\}_{j=0}^{10} = \\
&= \{\omega^0, \omega^0, \omega^1, \omega^2, \omega^3, \omega^1, \omega^0, \omega^3, \omega^0, \omega^1, \omega^5\}; \\
&\{\zeta_4(j)\}_{j=0}^{10} = \\
&= \{\omega^0, \omega^0, \omega^2, \omega^2, \omega^3, \omega^1, \omega^0, \omega^3, \omega^0, \omega^1, \omega^5\}; \\
&\{\zeta_5(j)\}_{j=0}^{10} = \\
&= \{\omega^0, \omega^0, \omega^2, \omega^3, \omega^4, \omega^3, \omega^2, \omega^5, \omega^2, \omega^4, \omega^2\}; \\
&\{\zeta_6(j)\}_{j=0}^{10} = \\
&= \{\omega^0, \omega^0, \omega^2, \omega^3, \omega^4, \omega^3, \omega^2, \omega^5, \omega^3, \omega^4, \omega^2\}.
\end{aligned} \quad (8)$$

Here ω is an arbitrary primitive sixth root of the unity:

$$\omega \in \{\exp(i.2\pi l)/6;\ l = 1, 2, ..., 5\} . \quad (9)$$

The sequences (8) form 15 different pair. The absolute values of some of their CCF, calculated by the aforementioned software module, are depicted on Fig.1 and Fig.2, where the following notations are used:

- *Ruv* is the CCF of the sequences $\{\zeta_u(j)\}_{j=0}^{10}$ and $\{\zeta_v(j)\}_{j=0}^{10}$, $u = 1, 2, ..., 5,\ v = 2, ..., 6,\ u \ne v$;
- *r* is the time-shift.

Fig.1a

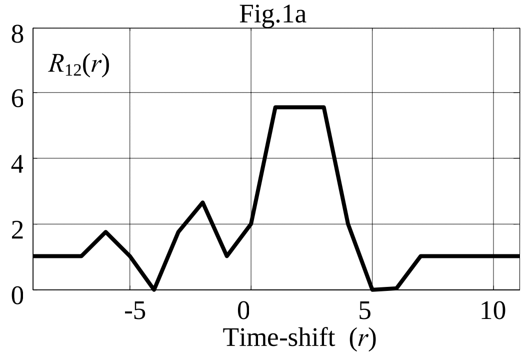

Fig.1b

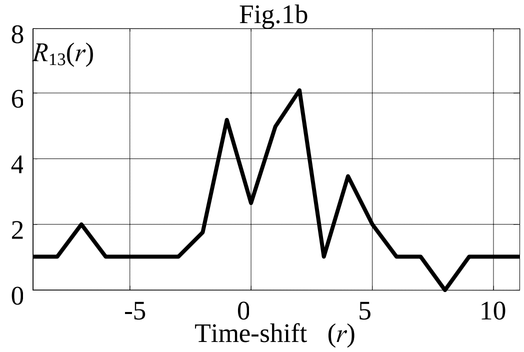

Fig. 1. The CCF of the sequences $\{\zeta_1(j)\}_{j=0}^{10}$ and $\{\zeta_2(j)\}_{j=0}^{10}$ (a), and sequences $\{\zeta_1(j)\}_{j=0}^{10}$ and $\{\zeta_3(j)\}_{j=0}^{10}$ (b).

As shown, the maximum level of the CCF of the pairs, comprising the sequences $\{\zeta_3(j)\}_{j=0}^{10}$, $\{\zeta_4(j)\}_{j=0}^{10}$ (Fig.2b) and $\{\zeta_5(j)\}_{j=0}^{10}$, $\{\zeta_6(j)\}_{j=0}^{10}$ exceeds too much the threshold $\sqrt{11} = 3{,}316$. Due to this reason, one sequence from the pair $\{\zeta_3(j)\}_{j=0}^{10}$, $\{\zeta_4(j)\}_{j=0}^{10}$ and one sequence from the pair $\{\zeta_5(j)\}_{j=0}^{10}$, $\{\zeta_6(j)\}_{j=0}^{10}$ must be excluded from the family (8). A precise analysis shows that the excluding of the sequences $\{\zeta_4(j)\}_{j=0}^{10}$ and $\{\zeta_6(j)\}_{j=0}^{10}$ is more appropriate. As a result, the family (8) reduces to a family, comprising four GBC: $\{\zeta_1(j)\}_{j=0}^{10}$, $\{\zeta_2(j)\}_{j=0}^{10}$, $\{\zeta_3(j)\}_{j=0}^{10}$ and $\{\zeta_5(j)\}_{j=0}^{10}$.

The maximum level of the CCF of the pairs of the new family is close to the minimal bound (threshold) $\sqrt{11} = 3{,}316$, and, consequently, it possesses optimal correlation properties.

Fig.2a

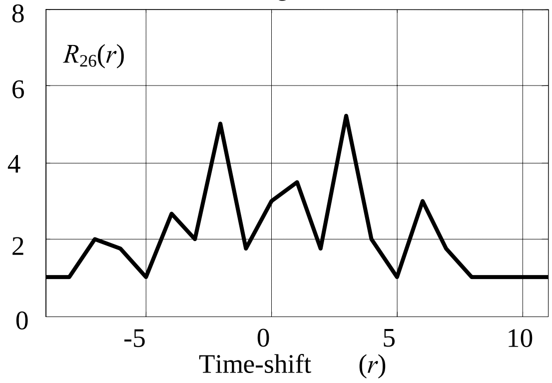

Fig.2b

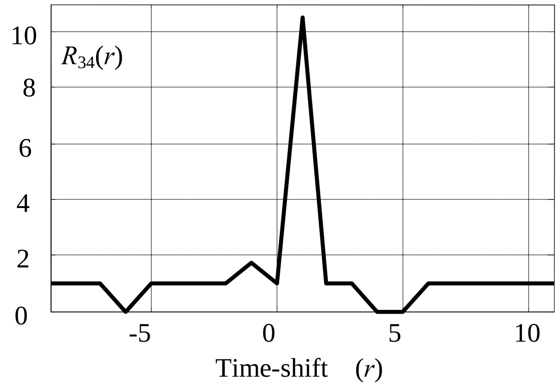

Fig. 2. The CCF of the sequences $\{\zeta_2(j)\}_{j=0}^{10}$ and $\{\zeta_6(j)\}_{j=0}^{10}$ (a), and sequences $\{\zeta_3(j)\}_{j=0}^{10}$ and $\{\zeta_4(j)\}_{j=0}^{10}$ (b).

IV. CONCLUSION

In the paper a survey of the cross-correlation properties of the generalized Barker codes (signals) is presented. This study is motivated by following reasons.

First, the generalized Barker codes are radio or audio signals with complex inner structure. Due to this reason their application in the radars and sonars leads to a significant enlargement of the range of distance measurements without loss of accuracy and ability to discriminate neighboring objects.

Second, the practical implementation of the generalized Barker codes is not complex, because only phase manipulation is involved. Just now this technique for spreading the spectrum of the signals is in very rapid progress, caused by its application in the mobile communications. As a result the hardware, forming and processing phase manipulated signals, is miniature, cost–effective and combines both very high productivity and low energy consumption.

The results, obtained in the paper, show that from the generalized Barker signals with given code length *n* and type *m* of PSK, families with low cross-correlation can be formed. As these families possess optimal correlation properties, they could be successfully used in many areas and technical applications, where the miniature size plays a crucial role - surveillance and control systems, autopilots of the cars, small scientific and spy airplanes, unmanned underwater vehicles and so on.

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