An Improved Method for Synthesis of Families of Costas Arrays

Valentin A. Mutkov, Nikolay R. Nikolov, Rumen G. Tsakov, and Liliya A. Staneva

***Abstract* — The present wireless communications must meet very strong conditions concerning the rate of data transmission, quality of service and electromagnetic compatibility. These requirements can be satisfy only by exploitation of families of complex radio signals, possessing special correlation properties. With regard, a method for synthesis of families of a class of ultra-wide band frequency hopping signals, called Costas arrays, is proposed in the paper. It is demonstrated that the method provides optimal correlation properties for all signals, belonging to a family of Costas arrays. The synthesized signals could be successfully applied in the wireless monitoring and control systems, ad hoc communication systems and so on.**

***Keywords* — Correlations, permutation matrices, signal synthesis.**

I. Introduction

THE communication and computer systems are in an unprecedented progress today. Indeed communication and computer systems have formed a common space where the quantity and quality of the offered services are growing very fast, which leads to the necessity of extreme optimal using of the connecting channels. This is a very hard technical problem, but the experience, obtained during the exploitation of the second and beyond generation wireless communication systems, shows that it can be successfully solved by usage of wide band signals, possessing special correlation properties. First of all, the side lobes of the auto-correlation functions (ACFs) of the signals should be as small as possible, because they determine the resolution of multiple copies of a signal, passed to the receiver through different paths. The fading of the received signals, caused by multipath spreading of the waves, is called self–interference (SI) [1], [2]. Second, the cross-correlation functions (CCFs) of all pairs of signals of a communication system should be close to zero. This condition allows large number of asynchronous users to share a common channel with an admissible level of the mutual interferences (or multi access interferences – MAIs).

Due to the importance of the signals with optimal correlation properties they have been extensively studied since the 1950s of the twentieth century [1]-[7]. Despite of the taken efforts, a few families with these abilities are known at present [6], [7]. With regard two modifications of a method for synthesis of the so-named Costas arrays are suggested in the paper. The interest to this problem follows from the fact that the Costas arrays guarantee high level of resistance to SI in very severe conditions of the wave spreading.

The paper is organized as follows. First, the basics of the Costas arrays are recalled. After that, two modifications of a method for synthesis of Costas arrays are presented. At the end, the areas of application of the results, obtained in the paper, are summarized.

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II. Basics of the Costas Arrays

In the 1960s Dr John P. Costas (USA) was puzzled by the poor practical performance of sonar systems [3]. He discovered that the rapidly time-varying channel made coherent processing inappropriate. As a consequence in the period 1962-1964 he set out to design a class of frequency-hopped waveforms, applicable in so severe conditions of the wave spreading. Most importantly, Costas was interested in waveforms with ideal (thumb-tack) shape of the auto-ambiguity correlation function (AACF). Practically simultaneously with Dr Costas and (may be) independently in the former USSR, the above problem was studied by Dr L. E. Varakin and Dr V. N. Vlasov [2]. The work of the Dr Costas became much popularity and due to this reason, the frequency-hopped waveforms, synthesized for usage in the sonar systems, were named Costas arrays [4]-[7].

The Costas arrays are complex signals formed by frequency manipulation (FM). Actually, they are a train of elementary pulses (chips) with different carrier frequency and can be mathematically described as follows:

$$v(t)=\sum_{j=1}^{n}U_m.u_0[t-(j-1).\tau_0]\times \times\cos\{2\pi[f_0+(d_j-1).\Delta f].t\} . \quad (1)$$

Here:

- U_m is the amplitude of the elementary pulses;
- f_0 is the basic carrier frequency of the signal;
- $u_0(t)=\begin{cases}1, & 0\le t\le\tau_0,\\ 0, & t<0\cup t>\tau_0,\end{cases}$
- τ_0 is the duration of the elementary pulses;
- Δf is the step of the variation of the frequency of the elementary pulses;

- $\{d_j\}_{j=1}^{n} = \{d_1, d_2, ..., d_n\}$ is a permutation of the integer numbers $\{1, 2, ..., n\}$; it defines the law of the FM and provides the thumb-tack shape of the AACF of the signal;
- n is the length of the train of elementary pulses, forming the Costas Array.

Often a Costas array can be described as a sequence of complex amplitudes of elementary signals:

$$V(t) = \sum_{j=1}^{n} U_m . \zeta(j) . u_0(t - j.\tau_0), \qquad (2)$$

where $\{\zeta(j)\}_{j=1}^{n}$ is the set of complex amplitudes of the elementary pulses:

$$\zeta(j) = \exp[i.2\pi.(d_j - 1).\Delta f], j = 0,1,...,n-1\}, i = \sqrt{-1} \qquad (3)$$

For simplicity (but without loss of generality) in the rest part of the paper it will be substituted in (1) and (2):

$$U_m = 1[V]. \qquad (4)$$

As a result of (4), the energy of the signals, studied in the paper, will be:

$$E = \frac{U_m}{2} n.\tau_0 [W] = \frac{n}{2} .\tau_0 [W]. \qquad (5)$$

In [3] a precise, but complex, expression for the AACFs of the Costas arrays is evaluated. In order to simplify the synthesis of the Costas arrays, the most of authors prefer to use the so-named matrix representation. According to it, the permutation $\{d_j\}_{j=1}^{n}$, presenting the law of the FM of a Costas array, is described as a quadrate matrix with n columns (time slots) and n rows (frequency bins), satisfying the following conditions:

A1. There are n dots, one in each row and one in each column (this is the permutation matrix constraint);

A2. No two of the line segments between the dots of all possible $\binom{n}{2}$ pairs have the same length and slope (this is the ideal AACF constraint).

These conditions are clarified on Fig. 1, where a Costas array with $n = 5$ and $\{d_j\}_{j=1}^{n} = \{3, 1, 5, 2, 4\}$ is depicted.

According to (1), the carrier frequency of the j-th elementary pulse is

$$f_j = f_0 + (d_j - 1).\Delta f, \quad j = 1, 2, ..., n, \qquad (6)$$

$$f_0 >> (n-1).\Delta f. \qquad (7)$$

It should be emphasized that the carrier frequencies (6) are spaced by the interval

$$\Delta f = \tau_0^{-1}, \qquad (8)$$

which provides the so-named orthogonal division of the carriers. This fact is shown on Fig.1 by a hatching of the small squares, located in the d_j-th row and j-th column.

Indeed the main part of the spectrum of the elementary pulses of the studied Costas array is concentrated in them.

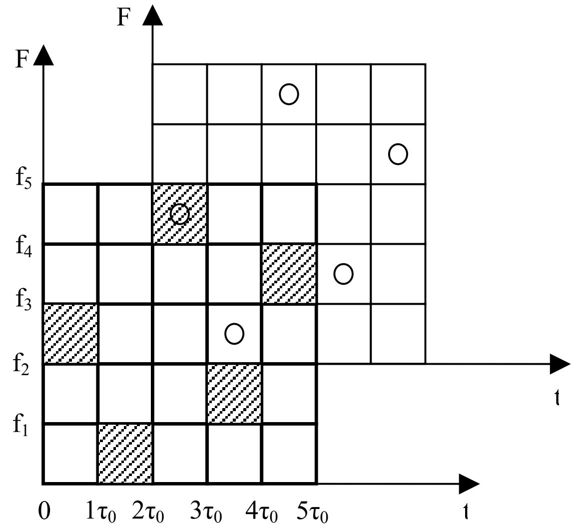

Fig. 1. The matrix representation of the Costas arrays.

In the process of communication, the carrier frequencies of the signals are changed by the so-named Doppler Effect, caused by the motion of the correspondents relatively one to other. More specifically, if f_s is the carrier sequence of the transmitted (sent) signal then the carrier sequence f_r of the received signal is:

$$f_r = (1 \pm k\frac{v_r}{v_0}).f_s. \qquad (9)$$

Here v_r is the velocity of the radial motion of the transmitter relatively to the receiver, v_0 is the velocity of the spreading of the waves, $k = 2$ in cases of radar or sonar systems and $k = 1$ for the usual communication systems. The sign is "plus" in case of convergence of the correspondents and is "minus" otherwise.

The aim of the receiver is to find simultaneously both the Doppler-shift of the carrier frequency and the time-shift of the received signal respectively to an etalon of the expected signal. In order to reach this aim [1], [2], the receiver performs the so-named correlation processing of the received signals, because this procedure maximizes the signal-to-noise ratio in the presence of white noise. As known [1], [2], the correlation processing is a calculation the CCF of the received signal and the etalon of the expected signal. Here it should be emphasized on following facts.

First, the AACF of a signal reduces to the ACF of the signal in case of a zero Doppler-shift and to the CCFs of the signal and its frequency shifted copies otherwise. Hence, the AACF of a signal is a generalization of its ACF.

Second, the receiver can perform successfully its role only if the differences among the used (etalon) signal and its frequency and time-shifted copies are as significant as possible. In other words, the AACFs of the used signals must be similar to a delta pulse (i.e. must have a thumb-tack shape).

Third, in case of the Costas arrays it is appropriate the frequency and time-shifts of the signals to be measured by the units Δf and τ_0 respectively.

The above facts are clarified on Fig.1, where a frequency and time shifted copy of the etalon Costas is depicted as a permutation matrix, filled with small circles. The circles designate the carrier frequencies of the elementary pulses of the copy of the signal. For simplicity on Fig.1 the shifts of the copy are chosen to be $2.\Delta f$ and $2.\tau_0$ respectively.

Accounting the matrix representation of the Costas arrays (Fig.1), it is not hard to conclude that the maximal contrast between the etalon signal and all its frequency and time shifted copies occurs in the case when only one circle coincides with a hatch. This is possible if all radius-vectors, formed by pairs of dots on the matrix representation of the Costas array, differ one from other by length or slope.

Due to their valuable correlation properties, the Costas arrays have been studied by many authors [3], [6], [7]. It is found that despite of usage of sophisticated computational methods for searching [7], the finding of Costas arrays remains a very hard task. Fortunately, at present several constructive methods for synthesis of Costas arrays are known [1], [4]-[7]. They allow the creating of Costas arrays with an arbitrary large size (length) *n*. With regard to these reasons the synthesis of families of Costas arrays, possessing both optimal auto-correlation and cross-correlation properties, has great practical importance. This problem is studied in more detail in the next part of the paper.

III. An Improved Method for Synthesis of Costas Arrays

The mentioned above constructive methods for synthesis of Costas arrays exploit the features of the finite algebraic fields (i. e. Galois Fields – GF). The most general of these methods are the following [4], [5], [7].

Method 1 (***Welch's method***): Given prime p and an arbitrary primitive element α of the GF(p), the sequence:

$$\{d_j\}_{j=1}^{p-1} = \{\alpha^1, \alpha^2, ..., \alpha^{p-1}\} \quad (10)$$

is a Costas array with length $n = p-1$.

Method 2 (***Golomb's method***): Given a prime power $q = p^m$ and arbitrary primitive elements α and β of the GF(q), the sequence:

$$\{d_j\}_{j=1}^{q-2} = \\ = \{\log_\beta(1-\alpha^1), \log_\beta(1-\alpha^2), ..., \log_\beta(1-\alpha^{q-2})\} \quad (11)$$

is a Costas array with length $n = q-2$.

It is known that every finite algebraic field has exactly

$$N = \varphi(q-1) \quad (12)$$

primitive elements (roots). Here $\varphi(.)$ is the so-named Euler's function.

Now the following abilities of the above methods should be taken in consideration.

First, the Costas arrays, obtained by the constructions (10) and (11) have complex pseudo-random structure, which is typical in general for all signals with optimal correlation properties [1]-[3], [6], [7].

Second, if the construction (10) is used with two different primitive elements α and β, the obtained permutations $\{d_{\alpha j}\}_{j=1}^{n}$ and $\{d_{\beta j}\}_{j=1}^{n}$ will coincide in only one position. Indeed, let the sequences $\{d_{\alpha j}\}_{j=1}^{n}$ and $\{d_{\beta j}\}_{j=1}^{n}$ are created by the Method 1, and

$$d_{\alpha j} = d_{\beta j} \quad (13)$$

for some j, $j \in \{1, 2, ..., p=1\}$. Then from (10) follows

$$\alpha^j = \beta^j, \quad (14)$$

which is possible only for $j = p-1$. Analogously it is not hard to see, that if the construction (11) is used with two different pairs of primitive elements (α, β) and (α, γ), the obtained permutations $\{d_{\alpha\beta j}\}_{j=1}^{n}$ and $\{d_{\alpha\gamma j}\}_{j=1}^{n}$ will not coincide.

The above conclusions lead to the following ***Improved Method for Synthesis of Families of Costas Arrays***.

Step 1) For given *n*, satisfying $n = p-1$ or $n = p^m - 2$, *p* prime, a no empty family of Costas arrays is generated, according to the Method 1 or Method 2.

Step 2) The cross-correlation properties of the Costas arrays, belonging to the above family, are examined by a software module, created by means of the MATLAB.

Step 3) The family of Costas arrays, created during the previous step, is separated in all possible pairs. After that the ambiguity cross-correlation functions (ACCFs) of the pairs are calculated.

Step 4) All Costas arrays which form pairs with maximal level of the CCF exceeding the threshold $\sqrt{n}$ are rejected.

The value $\sqrt{n}$ is chosen as an acceptable maximum level of the ACCF of the pairs of Costas arrays because the minimal bounds of the maximal level of the CCFs of a family of signals, is [8]:

$$L_{\min\max} \geq n.\sqrt{\frac{1-(1/N)}{n-(1/N)}}\,[V] \approx \sqrt{n}\,[V]. \quad (15)$$

Here $L_{\min\max}$ is the minimal maximum of the level of the CCFs and *N* is the size of the family of signals.

The results of the exploring of the Improved Method for Synthesis of Families of Costas Arrays, will be illustrated for $p = 11$, $n = 10$. In this case, according to (12), $N = 4$ primitive elements exist. Due to this reason, the usage of the Method 1 gives a family of Costas arrays with 4 members, which form 6 pairs of different Costas arrays. A general view of the AACF of these pairs of signals is presented on Fig.2, whereas on Fig.3 their maximal levels are shown (*R*uv is the maximal level of the ACCF of the *u*-th and *v*-th sequences).

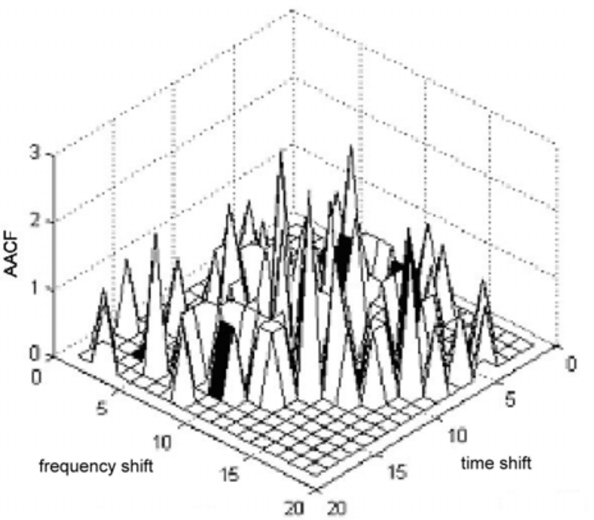

Fig. 2. The general view of the AACFs of the Costas arrays with length n=10.

Figures 2 and 3 demonstrate that the maximal levels of the ACCFs of the Costas signals, belonging to the considered family, do not exceed the threshold $\sqrt{11} = 3{,}316$, and, consequently, the created family possesses optimal correlation properties.

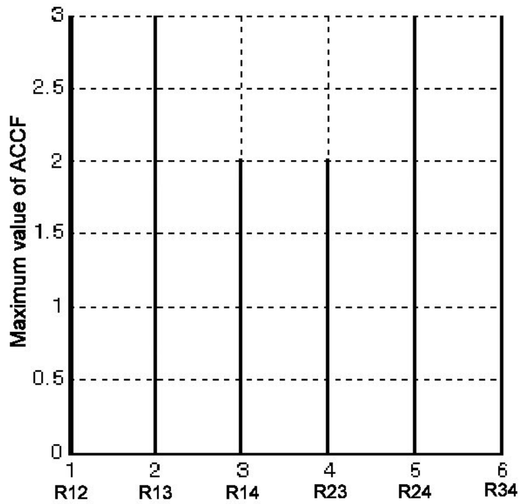

Fig. 3. The maximal levels of the ACCFs of the pairs of Costas arrays with length n=10.

Here it should be stressed that the condition (8) has a crucial role for the Costas arrays. A more precise analysis shows that it takes place only for the sonar systems, because only for these systems the magnitude of the Doppler shift of the carrier frequency of the signal is bigger than Δf. As in the high frequency radio-systems the Doppler-shift is many times smaller than Δf, the frequency shifts can be ignored during the analysis of the matrix representation of the signals of high-frequency radio-systems.

Due to this reason the forth step of the above Improved Method for Synthesis of Families of Costas Arrays can be excluded. Moreover, it can be modified as follows:

Step 1) For given n, a permutation matrix is generated. This can be done by constructions (10), (11) or any computational method as presented in [7], [9], [10].

Step 2) Let $\{d_j\}_{j=1}^{n} = \{d_1, d_2, ..., d_n\}$ is the permutation, generated during the previous step and b is an element, co-prime to n. Then the family of permutations:

$$\{d(l)_j\}_{j=1}^{n} = b^l \{d_1, d_2, ..., d_n\},\ l = 1, ..., \eta - 1 \quad (16)$$

In (16) η is the multiplicative order of b modulo n (i.e. η is the smallest integer with the property $b^\eta \equiv 1 \bmod n$).

The verifying of the Step 2 of ***the Modified Method for Synthesis of Families of Costas Arrays*** is not hard and is left.

IV. Conclusion

In the paper two modifications of a method for synthesis of families of Costas arrays are suggested. This work is motivated by following reasons.

First, the Costas arrays are signals with complex inner structure. As a result their application in the sonars leads to a significant enlargement of the range of distance measurements without loss of accuracy and ability to discriminate neighboring objects.

Second, the practical implementation of the Costas arrays is more complex than the implementation of some other types of complex signals (for example Barker signals, sequences with maximal length and so on) [1], [2]. Anyway, the families of the Costas arrays, which can be formed by the suggested in the paper methods, ensure the proper work of the communication devices in very severe conditions of the spreading of waves. Consequently, these families can be successfully applied in the systems, where the functional reliability has crucial importance.

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