Flexible Time-Frequency Decomposition in Subband Equalization

Vladimir V. Vityazev, *Member, IEEE*, Alexander Y. Linovich, *Member, IEEE*

***Abstract* — In this paper we review different methods of subband equalization based on oversampled filterbanks and present some new results for subband adaptive systems. Nowadays, there are a lot of applications, which use subband adaptive filters (SAFs). However, SAFs are still less well understood and studied compared to such mature and acknowledged technologies as OFDM or well-known least-mean-square time-domain algorithms. The advantages and drawbacks of multirate algorithms realized in SAFs over conventional adaptive algorithms like least-mean-square (LMS) and recursive least-mean-square (RLS) as well as over fast algorithms, like fast LMS, are considered below. The novel approach concerned with flexible time-frequency decomposition is also investigated. The main attention is focused on the oversampled SAFs both with real-valued and with complex-valued filter taps because of these SAFs let avoid aliasing of decimation by proper design of the analysis and synthesis filterbanks. In the last part of this paper it is shown that under certain circumstances SAF with flexible structure can be considered as the best choice in the trade-off between performance, computational complexity and processing delay.**

I. Introduction

Multirate algorithms used in subband adaptive systems are very attractive to be realized both in different modern communication systems and in the novel techniques will be emerged in perspective as well. On the one hand, filterbank (FB) multicarrier modulation is considered as a natural candidate for spectrum sensing and data communication in cognitive radio systems [1]. On the other hand, a subband adaptive filter (SAF) can substitute conventional equalizers in voice-band modems specified by Recommendations V.34 and V.90/V.92 (ITU-T), which are usually based on the least-mean-square (LMS), recursive least-mean-square (RLS), and fast LMS algorithms.

Most papers, published in the last several years, deal with FB multicarrier communication systems: filtered multitone (FMT) and cosine-modulated multitone (CMT). These technologies promise to be very powerful and, hence, significant research is necessary for their deployment in the near future. All multicarrier systems require the implementation of matched FBs in the transmitter and in the receiver simultaneously, that should be included in the Recommendation.

V. V. Vityazev, Ryazan State Radio Engineering University, Russia (tel: +74912961095; e-mail: tor@rgrta.ryazan.ru).

A. Y. Linovich, Ryazan State Radio Engineering University, Russia (tel: +74912961095; e-mail: tor@rgrta.ryazan.ru).

Unlike the multicarrier techniques, Recommendations of different single-carrier systems do not provide for implementation of FBs evidently, e.g. voice-band modems. The necessity of data rate increasing to approach channel capacity in the single-carrier applications require to devise new methods that are more efficient then conventional equalization algorithms. It should be noted an outstanding advantage of SAFs in single-carrier systems concerned with arbitrary choice of FB's structure. Some ideas of this paper were discussed more in more detail in [2], [3].

This paper is organized as follows. General information on SAFs with oversampled FBs is included in Section II. Section III shows simulation results. Our proposed flexible structure is described in Section IV. Then this paper is summarized in Section V.

II. System model

The general statement of equalization problem is formulated in [4], [5]. The channel is supplied with an adaptive filter. Ideally, this filter should compensate distortions appeared in the transmitted signal as a result of non-ideal channel frequency response. Theoretically, the series connection of the channel and the adaptive filter tends to be equivalent to the delay unit. The performance criterion is a power minimization of the error signal $e[n]$ that is defined as a difference between the desired signal $d[n]$ and the recovered waveform $y[n]$ derived from the received signal $x[n]$ by means of filtering.

The conventional time-domain adaptive algorithms (LMS, RLS, and their modifications) are generally applied in adaptive filters. Sometimes, when a high order adaptive filter is required, fast algorithms based on fast Fourier transform are used, e.g. fast LMS, which provide significant computational savings.

More complicated and at the same time more flexible adaptive structure can be designed using multirate algorithms. Detailed description of multirate signal processing can be found in [6]–[8]. The main operations of multirate signal processing are decimation and interpolation. In order to apply these operations, it is necessary to divide the input signal $x[n]$ into several frequency bands previously. The resulting structure called subband adaptive filter (SAF) is considered in many papers. There are certainly many different realizations of SAFs and so the most typical one [5] is shown in Fig. 1. Instead of one adaptive filter, the SAF's structure includes two and more adaptive filters (for instance, K adaptive filters in Fig. 1). The signal processing is performed in subchannels at the lower sampling rate; therefore filters' orders can be decreased about the same times. Here, K is the number of subchannels, $H_i(z)$ and $F_i(z)$ are the filters of the analysis and synthesis FBs, respectively ($i = 1,\ldots K$), M_i is a decimation/interpolation ratio in the i-th subchannel, and $\hat{W}_i\left(z^{M_i}\right)$ is an adaptive filter of the i-th subchannel operating at M_i times lower sampling rate. In subchannels any adaptive algorithms can be applied including subband adaptive filtering too. The main advantage of SAFs is a great decrease in computational complexity due to multirate signal processing. In the best case, when there are K equal subbands and all decimation ratios are also equal to K (oversampling ratio equals to 100%), the computational savings are maximal. Unfortunately, these critically sampled SAFs suffer from spectrum aliasing caused by non-ideality of FB's frequency responses. This is the main reason why only oversampled SAFs are considered in the further part of this paper.

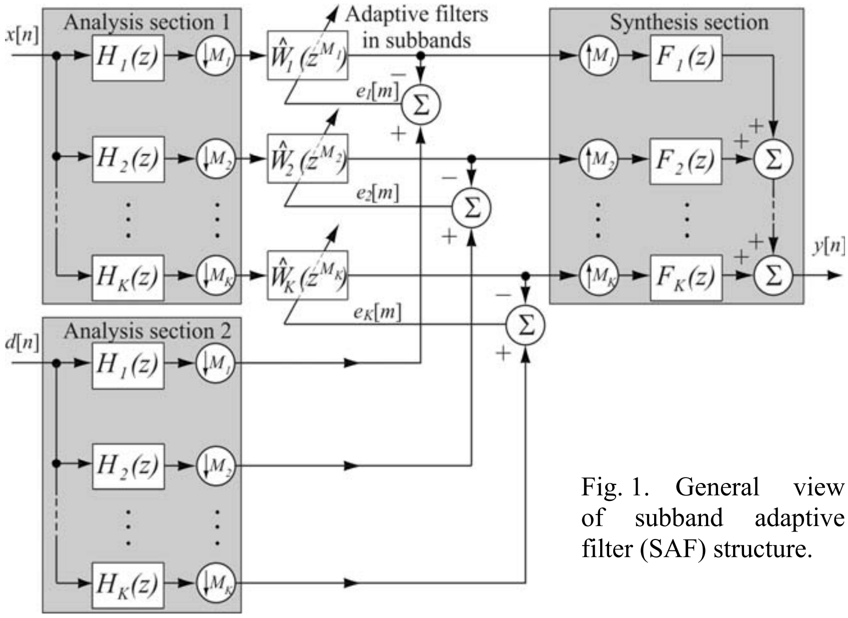


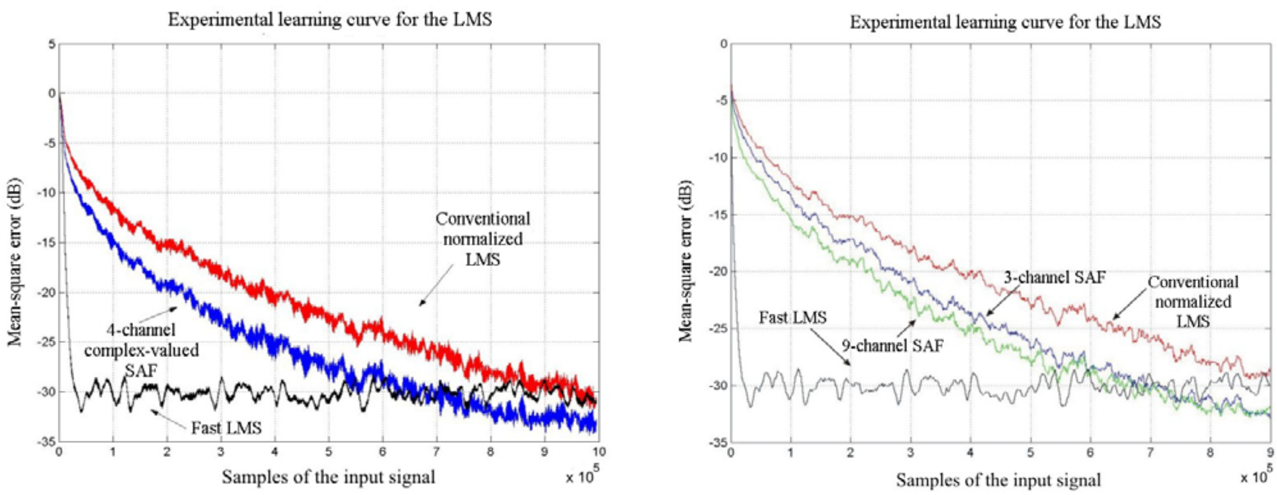
Fig. 1. General view of subband adaptive filter (SAF) structure.

III. SIMULATONS FOR SAFS SINGLESTAGE SUBBAND DECOMPOSITION

We have investigated different SAFs with oversampled FBs. Simulation results for LMS-SAFs are shown in Fig. 2. These results correspond to adaptive structures with the same equivalent fullband lengths, i.e., residual errors for all this structures are equal. It is obvious that increasing of the subband number results in the adaptive process acceleration. However, all LMS-SAFs yield to the fast LMS in speed.

Conversely, an adverse effect occurs when RLS-SAFs are investigated (not shown here): conventional RLS is the fastest algorithm. Nevertheless, subband adaptive structures provide considerable computational savings in comparison with LMS and especially with RLS algorithms.

A principle of oversampled SAF design rests on excluding of the spectrum aliasing caused by decimation and further interpolation [9]–[11]. For instance, in 3-channel SAF an input signal has to be decomposed in the analysis subsystem on 3 separate components: the low-frequency and high-frequency analysis filters are mirror and their decimation ratio is 2, for band-pass filter the decimation ratio is 3 or more can be specified. Taking into account that spectrum distortions are appeared if frequency bands cross over special points with frequencies multiple of $1/M_i$, the following rules for critical



(a) A complex-valued SAF with 4 subchannels.

(b) Real-valued SAFs with 3 and 9 subchannels.

Fig. 2. Training curves for different LMS-SAFs. All of these SAFs are based on oversampled FBs with different oversampling ratios.

frequencies of the 3-channel SAF can be determined [10]:

$$\omega_1 < \frac{1}{2}, \quad (1)$$

$$\omega_{21} > \frac{1}{3}, \ \ \omega_{22} < \frac{2}{3}, \quad (2)$$

$$\omega_3 > \frac{1}{2}. \quad (3)$$

If the decimation ratio of band-pass channel is equal to 5 or more, the second equation should be corrected with appropriate special frequencies multiple of $1/M_i$.

IV. Simulations for Multistage Subband Decomposition and Flexible SAFs

If it is necessary to use a high-order adaptive filter, the considerable computational savings can be achieved by involving additional stages of time-frequency decomposition. An example of the SAF with the simple 2-stage signal decomposition is shown in Fig. 3(a). This adaptive structure includes 2 stages of 3-channel decomposition and its equivalent fullband length is equal to 1200. Here, at the every elementary adaptive filter the order is specified. At the every analysis section output and every synthesis section input the decimation ratio is pointed out.

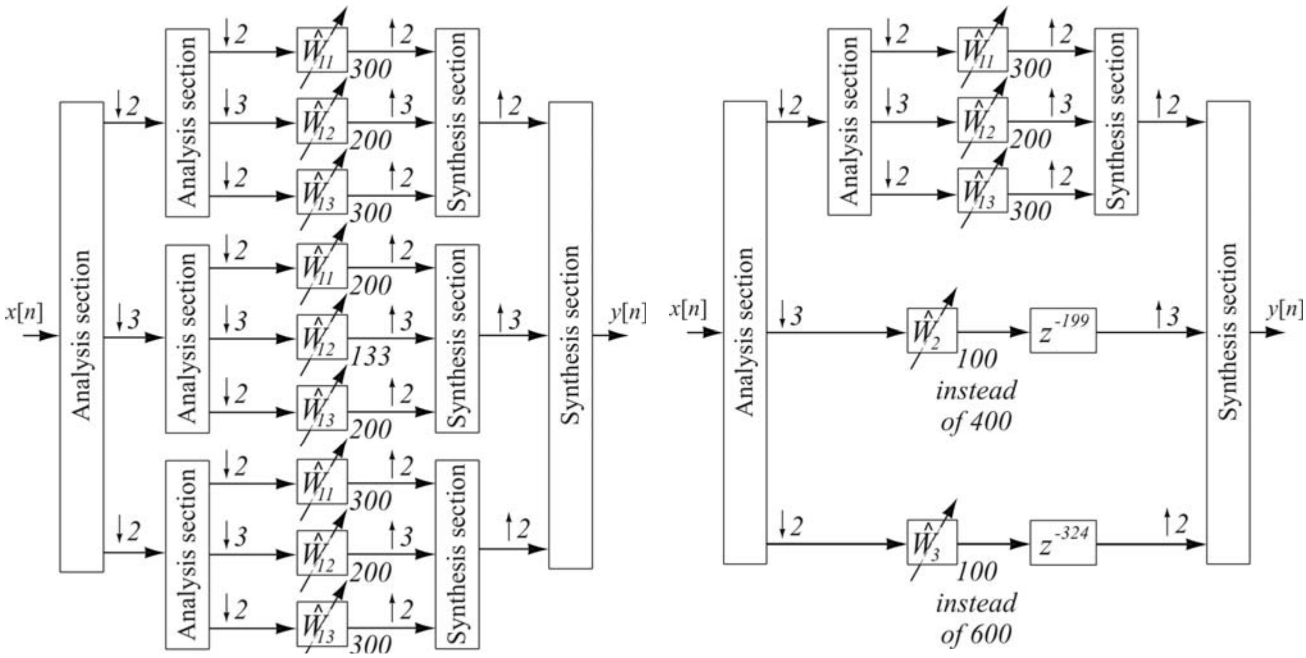
Sometimes we know, a priori or from investigation results, that the main distortions are concentrated in narrow frequency band (separate bands). In this case it is possible to use an amazing advantage of SAF. Let assume the most significant distortions are concentrated in the low-frequency band so they do not fall into the band-pass and high-pass channels of the SAF. Then we can keep the second stage of time-frequency decomposition in the low-pass band only, and remove the second-level analysis and synthesis sections from other channels. Moreover, we can decrease the number of filter taps in these channels as it is shown in Fig. 3(b).

This structure provides more flexible time-frequency decomposition of the input signal. It can be considered as the flexible SAF has different equivalent fullband lengths in different subbands. In Fig. 3(b), the SAF has following equivalent fullband lengths: 1200 in the low-frequency channel (the same as in Fig. 3(a)), 300 in the band-pass channel, and 200 in the high-frequency channel. Such values are sufficient to achieve almost the equal performance with the complete tree-type structure.

Now consider great computational savings due to subband adaptive filtering. Similar expressions for conventional LMS, RLS, and four instances of singlestage real-valued SAFs are presented in Table I. Here V_{as} is an approximate number of multiplications per input sample for analysis/synthesis subsystem, L_{eq} is an equivalent fullband length.

Table I
Computational complexities.

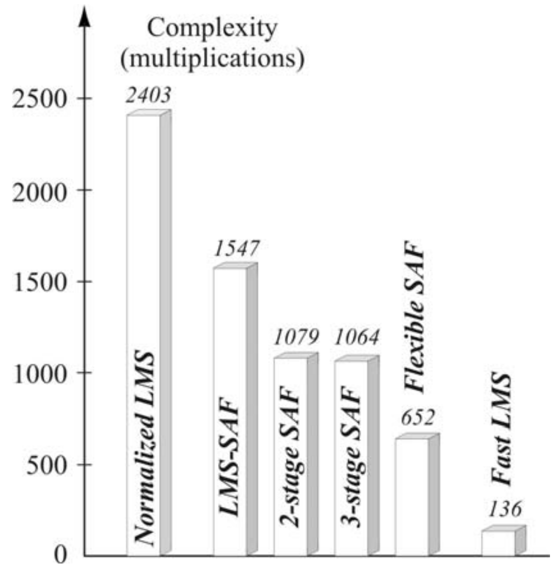
Structure type	Algorithm	
	LMS	RLS
Adaptive filter without decomposition	$2L_{eq}+3$	$2L_{eq}^2+4L_{eq}$
3-channel SAF ($V_{as}=76$)	$1{,}22L_{eq}+80$	$0{,}72L_{eq}^2+2{,}44L_{eq}+76$
9-channel SAF ($V_{as}=312$)	$0{,}85L_{eq}+316$	$0{,}28L_{eq}^2+1{,}32L_{eq}+312$



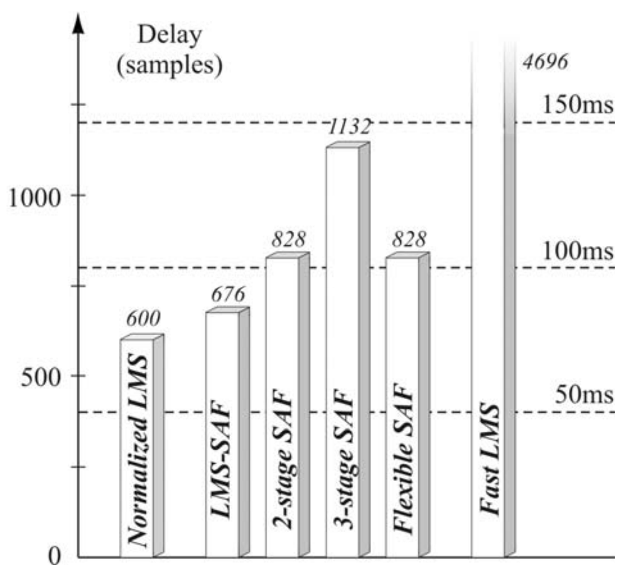
(a) SAF with the simple 2-stage signal decomposition: 3 × 3 = 9 channels.

(b) SAF constructed on the flexible principle. In this example it is assumed that the most severe distortions are concentrated in low-pass frequencies.

Fig. 3. Two structures: tree-type and flexible. Both structures provide equal performances (under specified conditions).



(a) Average quantity of multiplications per input sample.



(b) Approximate value of the processing delay.

Fig. 4. Comparison of LMS-based structures with the same equivalent fullband length equal to 1200. It is assumed the analysis/synthesis subsystem with $V_{as} = 76$ is applied in the SAFs.

The expressions for complexities of the multistage SAFs are cumbersome so it's difficult to summarize them in the Table I. All structures considered here have equivalent fullband length 1200. It is assumed the analysis/synthesis subsystem with $V_{as} = 76$ is applied in the SAFs.

Every additional stage of the composite analysis/synthesis subsystem increases matched computational costs. In this issue, there is no point in realizing 4-stage LMS-SAF for equivalent fullband length 1200 if the 3-channel analysis/synthesis subsystem with $V_{as} = 76$ is used.

In Fig. 4(b), time delays (in samples of the input signal) for real-time processing are compared. On the right side, a time delay for modems V.90/V.92 operated with the sampling frequency 8000 Hz is indicated.

These diagrams allow the following conclusion to make up. With little processing delay increase (about 38% in this example) and insignificant loss of accuracy, flexible SAF structures provide significant computational savings (about 3.7 times in this example) compared with the conventional LMS. It is obvious that for the RLS algorithm these savings are many times higher.

V. Conclusion

The results considered in this paper prove the efficiency of flexible subband equalization structures in single-carrier applications. Due to the multirate digital signal processing, SAFs provide significant computational savings compared with the conventional adaptive algorithms keeping almost the same performance of equalization. And vice versa, on the assumption of equal computational complexities for all equalization structures, SAFs enable to achieve better performance. The subband adaptive filtering is a very attractive technique in applications where there is a need to maximize the quality of equalization and reduce computational complexity. The amazing properties of SAFs are explained by the flexibility of time-frequency decomposition and by the multirate approach. However, the performance advantages and computational savings of SAFs are compensated by difficulties in their mathematical formulation. Therefore, the deployment of SAFs requires further research.

Reference

[1] B. Farhang-Boroujeny and R. Kempter, "Multicarrier Communication Techniques for Spectrum Sensing and Communication in Cognitive Radios," *IEEE Commun. Mag.*, vol. 46, no. 4, pp. 80–85, April 2008.

[2] A. Linovich, "Methods of time-frequency decomposition in applications deal with the inverse modeling problem", (in Russian), *Digital Signal Processing*, vol. 16, no. 3, pp. 28–37, 2005.

[3] V. Vityazev and A. Linovich, "A multirate approach to the inverse modeling problem: subband equalization," in Proc. XI Symposium AES "New Trends in Audio and Video," Bialystok, Poland, Sept. 20–22, 2006, vol. 1, pp. 389–394.

[4] B. Widrow and S.D. Stearns, *Adaptive Signal Processing*, NJ: Prentice-Hall, Englewood Cliffs, 1985.

[5] S. Haykin, *Adaptive Filter Theory*, 4th ed. NJ: Prentice-Hall, Upper Suddle River, 2002.

[6] R.E. Crochiere and L.R. Rabiner, *Multirate Digital Signal Processing*, NJ: Prentice-Hall, Englewood Cliffs, 1983.

[7] V.V. Vityazev, *Digital Frequency Selection of Signals*, (in Russian), Moscow: Radio and Communication, 1993.

[8] F.J. Harris, *Multirate Signal Processing for Communication Systems: Current Practice and Next Generation Techniques*, Prentice-Hall, 2004.

[9] Z. Cvetković and M. Vettely, "Oversampled filter banks," *IEEE Trans. Signal Process.*, vol. 46, no. 5, pp.1245–1255, May 1998.

[10] M. Harteneck, S. Weiss, and R.W. Stewart, "Design of near perfect reconstruction oversampled filter banks for subband adaptive filters", *IEEE Trans. Circuits Syst. II, Analog Digit. Signal Process.*, vol. 46, no. 8, pp. 1081–1085, Aug. 1999.

[11] P.P. Vaidyanathan, *Multirate Systems and Filter Banks*, NJ: Prentice-Hall, Englewood Cliffs, 1993.