Wave-based Modeling and Analysis of Microstrip Stub-line Structures

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Abstract – **The purpose of this paper is to show that the one-dimensional (1D) wave digital approach is suitable for modeling and analysis of stub-line structures. A stub-line structure, divided into uniform sections, can be efficiently modeled by wave digital network (WDN). Frequency response in WDN is obtained by direct analysis of formed block-based network. In the other hand, response can be calculated in the frequency or in the time domain directly from known network function in *z*-domain. The open T-resonator circuit, proving the response accuracy of the new technique, is given.**

Keywords – **Wave digital approach, wave digital networks, microstrip circuits, stub-line structures, T-resonators.**

I. INTRODUCTION

WAVE digital filters (WDFs) represent a class of digital filters with a particular interest. A detailed review of WDF theory is given in references [1]-[5]. A large variety of WDF-based techniques has been developed for a wide range of applications [2].

Well known theory of WDFs is used for modeling of the planar structures by wave digital elements. Recently, planar microwave stepped-impedance structures are modeled by use of the one-dimensional (1D) wave digital approach [5]-[16]. Standard wave digital elements (WDEs), such as delay, adder, multiplier and adaptor, are used in design of wave digital networks (WDNs) which represent one signal flow diagram.

There does not essentially exist just one type of microstrip structures, but a whole variety of quite distinct subclasses, each of which can again be divided into varies families, etc. For example, there exist stepped-impedance filters, stub-line structures and elliptic filters. This reflects the richness of WDNs, and the most appropriate one have to be chosen for structure at hand.

After an extensive search in the literature it was found an application of Advanced Design System (ADS [17]) to simulations of different microstrip structures based on their wave digital network representations [18]. The equivalent circuit model can be solved through a circuit simulator. The Kelly-Lochbaum implementation of adaptors (with many multiply elements) is used.

The main objective of this paper is to give an original and general method to characterize the behaviour of

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microstrip stub-line structures in both the frequency and time domains by use of the wave digital networks in MATLAB environment [19]-[20]. Why MATLAB? Wave digital approach into MATLAB provides easy simulation of microwave layouts such as microstrip structures of different geometries. Also, allows the user to get the fastest return out of 3D electromagnetic tool investments. Two-multiply models of the three-port adaptors are used here. The equivalent representation of this structure in MATLAB is much simple to operate than the one shown in [18].

Till now, stepped-impedance filters and nonuniform structures with linearly tapered lines are analyzed by use of suggested 1D approach [5]-[16]. Section II is devoted to some of the basic ideas of the 1D wave digital approach.

In view of their geometry, structures with stubs are also natural candidates for 1D approach. A new way for connecting segments in the structure with stubs is developed and represented in the Section III of this paper.

Response in WDN of stub-line structures can be found in two different ways as depicted in Section IV.

In Section V, a simulation validation of the proposed modeling and analysis approaches is provided by means of two examples of one open T-resonator circuit.

II. THE BASIC IDEA OF THE 1D WAVE DIGITAL APPROACH

The aim of this section is to provide a brief survey of the 1D wave digital approach in modeling and analysis of microstrip stepped-impedance structures given in the papers [6]-[16].

A nonuniform structure has to be divided into cascade-connected uniform transmission lines (UTL segments). If the complex microstrip structure comprises a certain number of uniform segments, each of them has to be represented as a cascade connection of a certain number of unit elements (UEs). This is because of their delays which vary from one another. A way of determination minimal number of UEs in WDN of complex microstrip structure is based on the given relative error as described in the papers [9], [14]. An important problem that needs to be carefully addressed is that of appropriate choice of a minimal section number in the model. This is very important because of its direct influence on the sampling frequency of that digital model, and on accuracy of the desired response. Also, it is very important to achieve a good compensation of the effects of identified step discontinuities. Many of the important results in modeling and analysis of microstrip stepped-impedance structures as well as step discontinuity are summarized in the paper [14] in order to emphasize their significance.

In a WDN, the connection of two UE with different port resistances is achieved by two-port series adaptors (TSAs), [3]. The WDN is a model of the microstrip structure where WDEs are used, and it is composed of two types of building blocks: UEs and TSAs.

Response in the WDN of stepped-impedance structure can be found in two different ways. In the first case, a block-diagram method for response calculation is used. WDN is formed directly in the block-oriented simulation programs such as Simulink toolbox of the MATLAB environment [19]-[20]. Response is obtained directly in the time domain, and Fourier transformation is used for frequency response calculation.

In the other hand, response can be calculated in the frequency or in the time domain directly from known network function in z-domain. The scattering parameters of WDN, S_{21} and S_{11}, are derived as rational functions of z-variable. A very simple transfer parameter approach for direct calculating of polynomial coefficients of these functions is described in the papers [7]-[8]. The algorithms are very easily implemented in the MATLAB environment. The analysis of the wave digital network is efficiently automated, which is inevitable when structures with large numbers of building blocks are to be dealt with.

The same procedures (shortly described above) can obviously also be applied to the modeling of the stub-line structures.

III. MODELING OF THE STUB-LINE STRUCTURE

The basic idea of the 1D wave digital approach is to treat the stub-line structure as a typical connection of separated uniform transmission lines (UTL segments). If 1D approach is used for analysis of structures with stubs, a new way for connecting UTL segments in that structures have to be developed [15]. In order to better explain the approach, one microstrip structure with a single stub, depicted in Fig. 1a, is used. This structure can not be observed as a simple cascade connection of UTL segments.

A wave digital network which corresponds to the segment connection shown in Fig. 1b is depicted in Fig. 2. Each uniform segment in the Fig. 1b can be approximated with one transmission line of characteristic impedance Z_{ck}, $k = 1,2,3$. Models of UTL segments are connected by use of one three-port parallel adaptor with port 2 being dependent (TPA). Adaptors are memoryless devices whose task is to perform transformations between pairs of wave variables that are referred to different levels of port resistance. In the symbolic representation of the three-port parallel adaptor [4] given in Fig. 3, they are shown explicitly the adaptor coefficients α_1 and α_2 next to the ports 1 and 3, respectively. These adaptor coefficients are

$$\alpha_j = \frac{2G_j}{G_1 + G_2 + G_3}, \qquad j = 1,2, \qquad (1)$$

where port conductances are

$$G_k = 1/Z_{ck}, \qquad k = 1,2,3. \qquad (2)$$

Segment UTL2 which represent open stub is connected to adaptor's dependent port 2. Each UTL segment of the
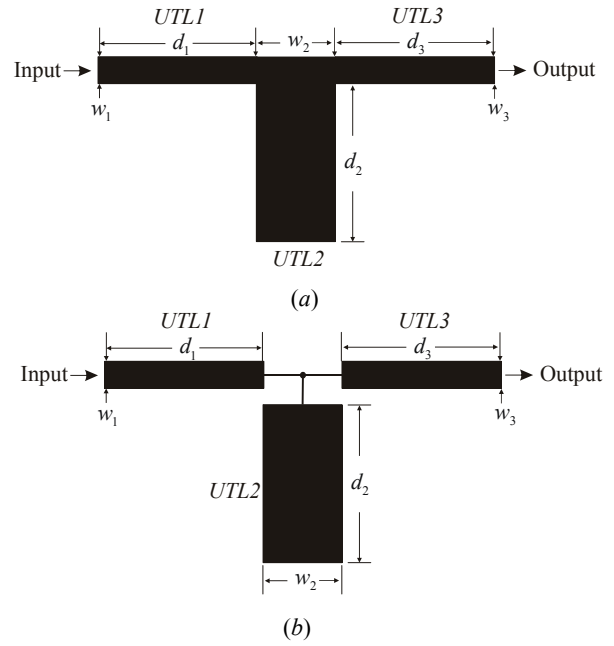

Fig. 1. Stub-line microstrip structure:
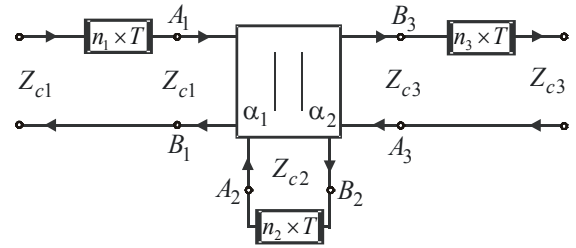(a) layout and (b) segment connections.

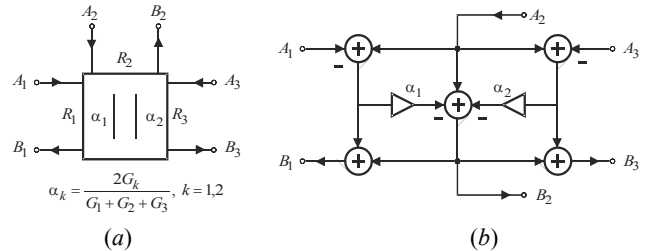Fig. 2. Wave digital network of the simple stub-line structure.

Fig. 3. Three-port parallel adaptor with port 2 being dependent: (a) symbol, and (b) WDE.

planar structure (transmission lines and stubs) modeled by n_k cascaded sections is assigned as $n_k \times T$ blocks, $k = 1,2,3$, in the Fig. 2.

IV. MODELING OF THE T-JUNCTION DISCONTINUITY

In this section, only discontinuities associated with the T-pattern resonator, such as open-end and T-junction, are considered. Since discontinuity dimensions are usually much smaller that the wavelength in a microstrip, they can be modeled by lumped-element equivalent ciruits.

An open-end discontinuity occurs frequently in a number of circuits such as resonators, matching stubs, filters, and microstrip antennas. A closed form expressions for calculating the excess length of transmission line [21] are used here.

A T-junction discontinuity is very offten used in the microstrip circuits, such as microstrip stubs, power dividers, etc [21]. For a symmetric T-junction structure, the equivalent network consists of the inductances in the series branches, and the parallel capacitance. The inductances and capacitance can be obtained by closed form expressions given in the papers [3]-[4].

A new modeling procedure of the equivalent circuit of T-junction discontinuity is based on decreasing line lengths. Corrections for T-junction effects are done as follows: the physical length of the series line which length is l_1 and width w_1 is decreased by the value $w_2 / 2$, i.e. it is decreased by the value of one half of width of the stub line. The physical length of the stub line with length l_2 and width w_2 is decreased by the value $w_1 / 2$, i.e. it is decreased by the value of one half of width of the series line.

This approach of modeling step discontinuity which involves changing line lengths is a very simple. Also, it is very acceptable because there are not new blocks here. In the case of the microstrip stub-line structure the proposed method reduces drastically the computation time while giving acceptable accuracy.

V. ANALYZING APPROACHES OF WAVE DIGITAL NETWORKS

The aim of this section is to describe possible analyzing approaches of stub-line structures, i.e. their wave digital network representations.

A. Block-based Analysis of Wave Digital Networks

The block-based approach in analysis of WDNs is the main topic of this subsection.

The formed Simulink model of WDN is depicted in Fig.4. The blocks **TLine_1**, **Stub_2** and **TLine_3** represent UTL segments. The blocks **ADP-S** and **ADP-L** represent two-port series adaptors, and block **ADP_T1S2T3** three-port adaptor. The two-port adaptors at the ends are used for matching source and load resistances to the rest of the WDN. In the symbolic representation of a two-port series adaptor [1]-[4] given in Fig. 5, it is shown explicitly the parameter α next to the port 1. The adaptor coefficients for the blocks **ADP-S** and **ADP-L** in the WDN are

$$\alpha_S = (R_S - Z_{c1})/(R_S + Z_{c1}), \qquad (3)$$
$$\alpha_L = (Z_{c2} - R_L)/(Z_{c2} + R_L).$$

Fig. 4. Simulink model of the structure.
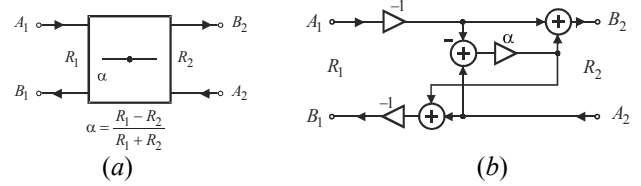
Fig. 5. Two-port series adaptor: (a) symbol, and (b) WDE.

B. Transfer Wave Matrix of Wave Digital Networks

The calculating of the scattering parameters of the known WDNs is discussed here. The theory of transfer wave matrix is the main topic of this subsection.

In general, a complex planar microstrip stub-line structure can be represented as M uniform segments connected in a typical way. The obtained wave digital network is composed of three types of building blocks: UEs, TSAa and TPAs.

A WDN with n_t UEs and $M-1$ three-port adaptors, pictured in Fig. 6, is analyzed here. This WDN is a two-port circuit having at each port an input and an output wave variable. Each UE is associated with its delay T, and port resistances R_k at either port, where $k = 1,2,...,M$. The simulation of connections between the three UTL segments (one of them is an open stub connected at dependent port) is achieved by three-port parallel adaptor. Each three-port adaptor is associated with its coefficients α_{k-1} and α_k, $k = 2,4,...,2(M-1)$, and three incident A_{k-1}, A_k and A_{k+1}, and three reflected B_{k-1}, B_k and B_{k+1} waves, where $k = 2,5,...,3(M-1)-1$. The incident wave A_0 is equal to voltage U_S of the source, and reflected wave B_m is equal to voltage $2U_L$ on the load. The first and the last two-port series adaptors are used for matching source and load resistances to the rest of the WDN.

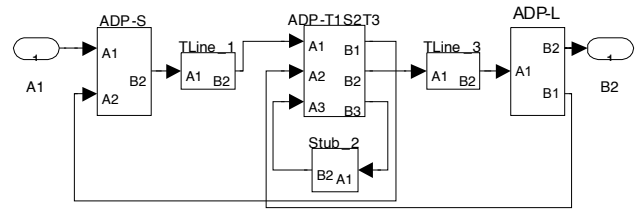The port impedances of the UEs in the $n_k \times T$ blocks are equal, which means that they can be directly cascade

Fig. 6. Ladder wave digital network of a planar microstrip stub-line structure.

connected (coefficients of two-port adaptors are zeros). The transfer wave matrix for a single UE is

$$T_{UE} = \begin{bmatrix} 1 & 0 \\ 0 & z \end{bmatrix}. \qquad (4)$$

Finally, the transfer wave matrix for one uniform segment which is modeled by n_k cascaded UEs is

$$T_{UE}^{n_k} = \underbrace{T_{UE} \times T_{UE} \times \cdots \times T_{UE}}_{n_k} = \begin{bmatrix} 1 & 0 \\ 0 & z^{n_k} \end{bmatrix} = \frac{1}{z^{-n_k}} \cdot \begin{bmatrix} z^{-n_k} & 0 \\ 0 & 1 \end{bmatrix} \qquad (5)$$

The kth three-port parallel adaptor with port 2 chosen as dependent port (Fig. 1) is described by set of equations

$$B_{k-1} = B_k + A_k - A_{k-1},$$
$$B_{k+1} = B_k + A_k - A_{k+1}, \qquad (6)$$
$$B_k = A_k + \alpha_{k-1} \cdot (A_{k-1} - A_k) + \alpha_k \cdot (A_{k+1} - A_k),$$

where the multiplier coefficients are

$$\alpha_{k-1} = \frac{2G_{k-1}}{G_{k-1} + G_k + G_{k+1}}, \qquad (7a)$$

and

$$\alpha_k = \frac{2G_{k+1}}{G_{k-1} + G_k + G_{k+1}}, \qquad (7b)$$

and G_{k-1}, G_k and G_{k+1} are the port conductances.

An open stub is connected to the port 2 being dependent and for the wave variables of this port can be written

$$A_k = z^{-n_k} \cdot B_k. \qquad (8)$$

The wave transfer matrix for the kth three-port adaptor, with an open stub connected on the dependent port 2, is obtained by use of relations (6)-(8) and it is

$$T_{\alpha_{k-1}\alpha_k}^{n_k} = \begin{bmatrix} T_{11}^{n_k} & T_{12}^{n_k} \\ T_{21}^{n_k} & T_{22}^{n_k} \end{bmatrix}, \qquad (9a)$$

where the matrix elements are

$$T_{11}^{n_k} = \frac{(\alpha_{k-1} + \alpha_k - 1) + z^{-n_k}}{\alpha_{k-1} \cdot (1 + z^{-n_k})}, \qquad (9b)$$

$$T_{12}^{n_k} = \frac{(\alpha_{k-1} - 1) - (\alpha_k - 1) \cdot z^{-n_k}}{\alpha_{k-1} \cdot (1 + z^{-n_k})}, \qquad (9c)$$

$$T_{21}^{n_k} = \frac{(1 - \alpha_k) + (\alpha_{k-1} - 1) \cdot z^{-n_k}}{\alpha_{k-1} \cdot (1 + z^{-n_k})}, \qquad (9d)$$

and

$$T_{22}^{n_k} = \frac{(\alpha_{k-1} + \alpha_k - 1) \cdot z^{-n_k} + 1}{\alpha_{k-1} \cdot (1 + z^{-n_k})}. \qquad (9e)$$

The complete transfer wave matrix T corresponding to the analyzed WDN is a product of the wave matrix of network building blocks as

$$T = T_{\alpha_s} \times T_{UE}^{n_1} \times T_{\alpha_1\alpha_2}^{n_2} \times T_{UE}^{n_3} \times \ldots \times T_{UE}^{n_{k-1}} \times T_{\alpha_{k-1}\alpha_k}^{n_k} \times$$
$$\times T_{UE}^{n_{k+1}} \times \ldots \times T_{\alpha_l}. \qquad (10)$$

Consider now the matrices of two-port series adaptors as follows

$$T_{\alpha_s} = \frac{1}{1-\alpha_s} \cdot \begin{bmatrix} -1 & \alpha_s \\ \alpha_s & -1 \end{bmatrix} = \frac{1}{1-\alpha_s} \cdot T_s, \qquad (11)$$

and

$$T_{\alpha_l} = \frac{1}{1-\alpha_l} \cdot \begin{bmatrix} -1 & \alpha_l \\ \alpha_l & -1 \end{bmatrix} = \frac{1}{1-\alpha_l} \cdot T_l, \qquad (12)$$

where adaptors' coefficients are

$$\alpha_s = \frac{R_s - R_1}{R_s + R_1}, \text{ and } \alpha_l = \frac{R_M - R_l}{R_M + R_l}, \qquad (13)$$

with the port resistances R_s, R_1, R_M, and R_l assigned as shown in Fig. 6.

The matrix of one uniform segment modeled with n_{2j-1} UEs can be written in the form

$$T_{UE}^{n_{2j-1}} = \frac{1}{z^{-n_{2j-1}}} \cdot \begin{bmatrix} z^{-n_{2j-1}} & 0 \\ 0 & 1 \end{bmatrix} = \frac{1}{z^{-n_{2j-1}}} \cdot T_{n_{2j-1}}, (14)$$

where is $j = 1,2,...,N_1$.

The matrix of three-port parallel adaptor with stub on dependent port 2 which is modeled by n_{2j} UEs is given in the form

$$T_{\alpha_{2j-1}\alpha_{2j}}^{n_{2j}} = \frac{1}{\alpha_{2j-1} \cdot (1 + z^{-n_{2j}})} \cdot T_{n_{2j}} =$$

$$= \frac{1}{\alpha_{2j-1} \cdot (1 + z^{-n_{2j}})} \cdot \begin{bmatrix} T_{11}^{n_{2j}} & T_{12}^{n_{2j}} \\ T_{21}^{n_{2j}} & T_{22}^{n_{2j}} \end{bmatrix}, \qquad (15a)$$

where the matrix elements are

$$T_{11}^{n_{2j}} = (\alpha_{2j-1} + \alpha_{2j} - 1) + z^{-n_{2j}}, \qquad (15b)$$

$$T_{12}^{n_{2j}} = (\alpha_{2j-1} - 1) - (\alpha_{2j} - 1) \cdot z^{-n_{2j}}, \qquad (15c)$$

$$T_{21}^{n_{2j}} = (1 - \alpha_{2j}) + (\alpha_{2j-1} - 1) \cdot z^{-n_{2j}}, \qquad (15d)$$

$$T_{22}^{n_{2j}} = (\alpha_{2j-1} + \alpha_{2j} - 1) \cdot z^{-n_{2j}} + 1, \qquad (15e)$$

and $j = 1,2,...,N_1$. The analyzed planar structures can have different number of UTL segments. The number N_1 depends on the number of segments in structure as

$$N_1 = \begin{cases} M/2 & \text{for } M \text{ being even,} \\ (M-1)/2 & \text{for } M \text{ being odd.} \end{cases} \qquad (16)$$

According to the relations (10)-(16), the polynomials can be written in the form

$$W_e(z) = (1-\alpha_s) \cdot (1-\alpha_l) \cdot \prod_{j=1}^{N_1} \left(\alpha_{2j-1} \cdot (1 + z^{-n_{2j}}) \cdot z^{-n_{2j-1}} \right)$$

$$\text{for even } M, \qquad (17)$$

or $\qquad W_o(z) = W_e(z) \cdot z^{-n_M}$, for odd M. $\qquad (18)$

The complete matrix can be represented in the form

$$T_e(z) = T_s \times \prod_{j=1}^{N_1} \left(T_{n_{2j-1}} \times T_{n_{2j}} \right) \times T_l, \quad \text{for even } M,$$

(19)

or

$$T_O(z) = T_s \times \prod_{j=1}^{N_1} \left(T_{n_{2j-1}} \times T_{n_{2j}} \right) \times T_{n_M} \times T_l, \text{ for odd } M,$$

(20)

where the matrix T_{n_M} corresponds to the last segment in the series branch.

Finally, the complete wave transfer matrix T due to the number of segment in the structure can be written in one of two forms

$$T = \frac{1}{W_e(z)} \cdot T_e(z), \text{ for even } M \quad (21)$$

or $\quad T = \frac{1}{W_o(z)} \cdot T_o(z), \text{ for odd } M. \quad (22)$

In other words, the complete wave transfer matrix can be written in the form of polynomials

$$T = \frac{1}{W_{e/o}(z)} \cdot T_{e/o}(z) = \frac{1}{W_{e/o}(z)} \cdot \begin{bmatrix} T_{e/o11}(z) & T_{e/o12}(z) \\ T_{e/o21}(z) & T_{e/o22}(z) \end{bmatrix} =$$
$$= \begin{bmatrix} T_{11}(z) & T_{12}(z) \\ T_{21}(z) & T_{22}(z) \end{bmatrix},$$

(23)

where index e/o corresponds to even (e) or odd (o) number M. The wave matrix elements are the rational polynomial functions of z^{-1}, and the elements are

$$T_{11}(z) = \frac{T_{e/o11}(z)}{W_{e/o}(z)} = \frac{1}{W_{e/o}(z)} \cdot \sum_{k=0}^{n} \left(z^{-k} \cdot T_{e/o22}(m-k) \right),$$

(24)

$$T_{12}(z) = \frac{T_{e/o12}(z)}{W_{e/o}(z)} = \frac{1}{W_{e/o}(z)} \cdot \sum_{k=0}^{n} \left(z^{-k} \cdot T_{e/o12}(k+1) \right),$$

(25)

$$T_{21}(z) = \frac{T_{e/o21}(z)}{W_{e/o}(z)} = \frac{1}{W_{e/o}(z)} \cdot \sum_{k=0}^{n} \left(z^{-k} \cdot T_{e/o12}(m-k) \right),$$

(26)

$$T_{22}(z) = \frac{T_{e/o22}(z)}{W_{e/o}(z)} = \frac{1}{W_{e/o}(z)} \cdot \sum_{k=0}^{n} \left(z^{-k} \cdot T_{e/o22}(k+1) \right).$$

(27)

The polynomial coefficients of the wave matrix elements depend only on adaptors' coefficients. According to relations (24)-(27), only two elements, $T_{12}(z)$, and $T_{22}(z)$, have to be calculated. If necessary, two other elements, $T_{11}(z)$, and $T_{21}(z)$, can be derived from relations (24) and (26).

Providing $A_m = 0$, the output response (forward voltage transmission coefficient) is

$$S_{21} = \frac{B_m}{A_0}\bigg|_{A_m=0} = \frac{W_{e/o}(z)}{T_{e/o22}(z)}, \quad (28)$$

and the input response (input reflection coefficient) is

$$\Gamma_0 = S_{11} = \frac{B_0}{A_0}\bigg|_{A_m=0} = \frac{T_{e/o12}(z)}{T_{e/o22}(z)}. \quad (29)$$

VI. EXAMPLES OF APPLICATION

In this section, a simple example of stub-line microstrip structures is analyzed by use of suggested 1D approach.

A. T-resonator Circuit - Example I

A microstrip structure with a single stub, depicted in Fig.1, is used for verification of the proposed method. The substrate dielectric constant is $\varepsilon_r = 2.32$ and the board thickness $h = 1.58\,mm$.

T-resonator circuit is approximated by connection of three uniform lines with parameters given in the Tables I and II. In order to absorb discontinuity effects new line lengths are counted. In this case, T-junction discontinuity is modeled by decreasing lengths of the line in the junction. The effect of the open stub is compensated by increasing length of the segment UTL2 [16]. According to the parameters given in the Tables I and II, it can be concluded that the number of transmission line is the same in both cases, but their physical length differ one another and beacuse of that their delays differ either.

TABLE I
TRANSMISSION LINE PARAMETERS WITHOUT MODELED
DISCONTINUITIES

nv	d [mm]	w [mm]	Zc [Ohm]	Tv [ps]
1	30.0000	4.7100	50.2540	139.8003
2	30.0000	15.7600	20.0016	145.0746
3	30.0000	4.7100	50.2540	139.8003

TABLE I
TRANSMISSION LINE PARAMETERS WITH MODELED
DISCONTINUITIES

nv	d [mm]	w [mm]	Zc [Ohm]	Tv [ps]
1	22.1200	4.7100	50.2540	103.0794
2	28.4681	15.7600	20.0016	137.6665
3	22.1200	4.7100	50.2540	103.0794

A minimal number of sections for the given error can be found as described in the papers [13]-[14] where multiple factor $q \geq 1$ is used. In the case where discontinuties are being modeled, for given error $n\_er = 0.01\%$, a total minimal number of sections in the observed WDN is $n_t = \sum_{k=1}^{3} n_k = 447$. For segments UTL1 and UTL3, a number of sections n_k is 134, and for segment UTL2 is 179. A total delay for the digital model of the structure is $T_t = n_t \cdot T_{\min} / q = 343.8254\,ps$, where $q = 134$ is a multiple factor and $T_{\min} = \min\{T_1, T_2, T_3\} = 103.0794\,ps$ is a minimum delay. A total real delay of the structure is $T_\Sigma = \sum_{k=1}^{3} T_k = 343.8544\,ps$. A sampling frequency of the digital model of the planar structure for the chosen minimal number of sections is $F_s = n_t / T_t = 1299.9687\,GHz$. In this case, a relative error of delay is $er = -0.008457\%$. Adaptor coefficients are $\alpha_S = -\alpha_L = -0.0025$ and $\alpha_1 = \alpha_2 = 0.4432$.

Response comparison for the proposed approach of modeling structrure with stub by use of 1D approach is

shown in Fig. 7. The $S_{21}[dB]$ results are similar in general. The WDN result shows only a small difference of the resonance frequency in comparison with the result of electromagnetic simulation in ADS [18].
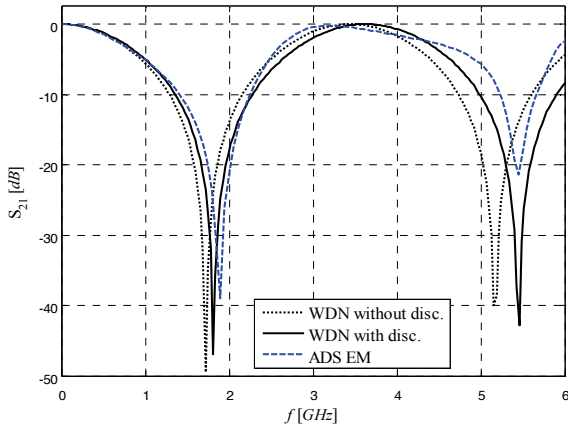

Fig. 7. Frequency response.

In this structure, a number of uniform section is odd ($M = 3$), and according to this, the forward voltage transmission coefficient in z-domain is

$$S_{21}(z) = W_o(z)/T_{o22}(z),$$

and, the output reflection coefficient

$$S_{22}(z) = -T_{o21}(z)/T_{o22}(z).$$

These coefficients are the rational functions, where the polynomials in numerators are

$$W_o(z) = 0.4432 + 0.4432 \cdot z^{-179},$$

$$T_{o21}(z) = -0.0025 + 0.5568 \cdot z^{-134} + 0.0003 \cdot z^{-179} -$$
$$- 0.0003 \cdot z^{-268} - 0.5568 \cdot z^{-313} + 0.0025 \cdot z^{-447},$$

and the polynomial in denominator is

$$T_{o22}(z) = 1 - 0.0028 \cdot z^{-134} - 0.1136 \cdot z^{-179} +$$
$$+ 7.2890 \cdot 10^{-7} \cdot z^{-268} + 0.0028 \cdot z^{-313} - 6.4180 \cdot 10^{-6} \cdot z^{-447}.$$

As previously stated in the subsection B of section V, it is quite enough to calculate the scattering parameters on the network output, $S_{21}(z)$, and $S_{22}(z)$. For the magnitudes of the parameters can be written $|S_{12}(z)| = |S_{21}(z)|$, and $|S_{11}(z)| = |S_{22}(z)|$. The phase characteristics differ from one another. Known network functions in z-domain can be used later as input data in some other simulations.

B. T-resonator Circuit - Example II

A microstrip structure with a single stub, depicted in Fig. 1, is used for verification of the proposed method. The structure is fabricated on CuFlon substrate with dielectric constant $\varepsilon_r = 2.17$ and width $h = 0.508\,mm$.

The S_{21} parameters of the structure at the frequencies from $300\,MHz$ to $6\,GHz$ are shown in Fig. 8. The transmission parameter was measured by a network analyser. A very good agreement between the simulated results was also obtained for many other structures of different shapes with T-junctions, which are not presented here. The measured result shows a small difference of the resonance frequency in comparison with the simulated result. This difference is caused by the microstrip T-

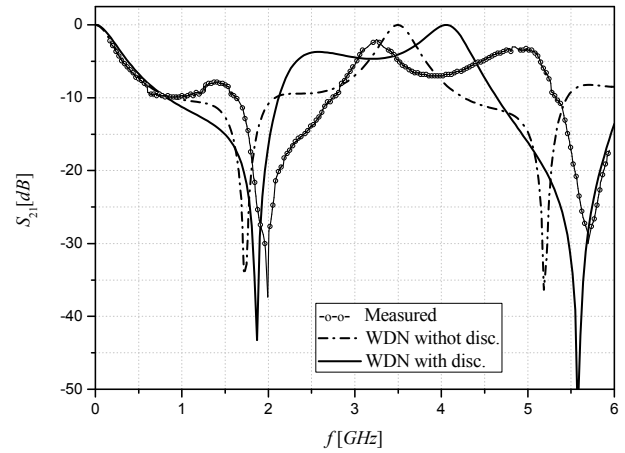junction and the open-end, which are not described accurate enough by the models.

Fig. 8. The measured and simulated transmission parameters of the open T-resonator circuit.

According to these findings, a simple 1D wave digital approach can produce results that are similar to the much more sophisticated methods. The simulation showed that the discontinuities of the microstrip T-resonator structure had a great effect on the resonance frequencies.

C. Comparison of Analysis Methods

The observed microstrip stub-line structure has been analyzed in this paper directly in the frequency and in the time domain. A new approach based on wave transfer parameters has been described. The proposed approach is implemented using MATLAB on a Pentium V personal computer operated on $2.4\,GHz$. Also, the purpose of this part is to compare the duration of two response calculation ways based on the results of their simulation runs.

Block-based WDN is formed directly in the Simulink toolbox of the MATLAB environment. Frequency response is obtained by direct analysis of formed block-based network. In that case, results are obtained in $2.1265\,s$. If network is analyzed here directly in the time domain, much more simulation time is required, and a memory problem is very frequent. A time for drawing WDN in the Simulink toolbox depends only on user's skills and it is not given here.

Frequency response from the known network function in z-domain can be found in two different ways. The first case starts with the response calculation directly in the time domain using differential equations. On the final stage Fourier transformation is used for frequency response calculation. A time of $4.2544\,s$ is needed for response calculation. In the second case, a time for a response calculation in the frequency domain directly is $0.0061\,s$. The results are showing significant difference between these two times. A great advantage of the second response calculation way is its computational efficiency, because all necessary calculations take place directly in the frequency domain.

This study focuses on a comparison of simulation times. As seen here, the simplest way for response calculation is direct analysis of WDN in the frequency domain and use of network transfer function in z-domain. This function

can easily be obtained from the wave transfer matrix. In that case, there is no high memory request and a very short time is needed for response calculation.

VI. CONCLUSION

The primary purposes of this paper are fourfold.

First, the 1D wave digital approach used for analysis of microstrip stepped-impedance structures is presented in a way that can be also used in analysis of stub-line structures. A new way for connection of identified UTL segments is described. This leads to a universal and effective procedure capable of solving a wide range of practical problems.

The second objective is to give the original and general methods to characterize the behaviour of microstrip stub-line structures in both the frequency and time domains. Firstly, a very simple block-diagram method of analysis of the WDN is used here. WDN is formed directly in the Simulink toolbox of the MATLAB environment. Frequency response is obtained by direct analysis of formed block-based wave digital network. Also, response is calculated in the frequency or in the time domain directly from known network function in z-domain. The wave digital principles combined with transfer parameter approach can be used advantageously to determine response of wave digital structure. This combination is a very efficient way for evaluating the scattering parameters of wave digital structures. A simple algorithm for coefficient calculation was derived. The described procedure can be implemented very effectively in any matrix oriented mathematical programs. Also, the study of discontinuities associated with the T-pattern resonator is a very important. The simplest modeling approach of the equivalent discontinuity network involves changing line lengths.

The third objective is to prove the accuracy of the proposed modeling and analyzing approaches. The broadband accuracy of the suggested procedures is validated by two examples realized in the microstrip line technique, such as open T-resonator circuit. The results of the analysis obtained by the WDN have shown a very good agreement with those obtained by other programs mentioned above. The results of one T-resonator are compared with measurements performed in the frequency range $300\,MHz$ - $6\,GHz$. A much better agreement is achieved by modeling discontinuities in the observed structure.

The last objective, but not the least, is to show computational efficiency of suggested approaches. A comparison the duration of two response calculation ways (block-diagram method and network functions) based on the results of their simulation runs is done. A great advantage of this method is its computational efficiency. This approach provides the fast structure simulation versus complex and time consuming 3D models.

Implementation of WDNs in analysis of microwave structures can be used by microwave engineers because of the associated simplicity and accuracy. It can be used for analyzing the transmission lines of various nonuniform shapes present in practice.

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