# Resource Allocation for Cognitive Relaying Network

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Abstract — In this paper, the problem of wireless resource allocation in OFDMA-based cognitive relaying network is investigated. The objective is maximizing the throughput of primary users in a system in which cognitive users act as a relay for primary users. In our proposed scheme, while primary users achieved desire rate, they dedicate some subcarrier for cognitive users. Considering the benefits of cognitive radio and cooperative relaying, the studied problem is formulated as nonlinear optimization problem. Finally, the performance of the proposed resource allocation algorithms is studied through simulation results which illustrate performance improvement of primary users.

Keywords — resource allocation, OFDMA, cognitive radio, cooprativ, optimazation

# I. INTRODUCTION

Recently, as a result of emergence of a variety of applications, the requirement for wireless services has been on raised rapidly. Today the spectrum is overcrowded and there are a few spaces available for future wireless applications [1].

The Federal Communications Commission (FCC) indicates that many portions of the licensed spectrum are not used efficiently [1]. In order to fully utilize the valuable spectrum more efficiently, Cognitive Radios (CR) concept was introduced by J. Mitola in his PhD dissertation in 2000 [2]. Cognitive users are allowed to reuse underutilized frequency bands if they avoid introducing interference to primary users by adjusting their transmission parameters.

Since cooperative communication provides multiuser diversity, it improves the performance of wireless networks. In cooperative communication, antennas of different users are shared to make virtual antenna array. Several cooperative strategies are proposed such as amplify-and forward (AF) protocol decode-and-forward (DF) protocol and coded cooperation (CC) protocol [3].

Due to flexibility orthogonal frequency division multiplexing (OFDM) in allocating radio resource, CR systems employ it commonly. The resource allocation for OFDM-based cognitive radio networks has been widely studied at physical (PHY) layer in the term of subcarrier, bit, and power allocation [4]-[5]. The algorithms proposed in [4] maximize the weighted sum of cognitive user rates under the constraint of multiple primary users' interference temperature. In [5], the authors proposed the optimal allocating of transmission time and power to minimize overlap between Primary Users (PU) and Secondary Users (SU).

Combinations of cognitive radio and cooperation concept are proposed in [6] in which cognitive users help primary users as a relay. The authors investigate the system performance from MAC layer point of view. Improving system throughput through the problem of resource allocating in cognitive relaying system has received little attention.

In this paper, we consider a scenario in which cognitive users help primary users send their data. Cognitive users apply amplify-and-forward (AF) protocol for cooperation in which the relay amplified signal received from source and transmits the amplified signal to the destination. We investigate uplink resource allocation for primary users in an OFDMA-based cognitive relaying network to maximize the throughput of primary users by nonlinear optimization problems with total transmit power constraint for primary users. Since cognitive users dedicate power for relaying primary users' data, we assume that if primary users achieve their desired rate, they try to vacate some subcarriers for the use of secondary users. It means that while we consider the desired rate of primary users, we have another constraint on the number of subcarriers used by primary users. If cooperative transmission with the constraint on the number of subcarrier doesn't surpass non cooperative scenario, primary users will return to non cooperative scenario. However, if cognitive users allocate enough power, this case will not happen and through the help of secondary users, throughput of the studied system will be better than non cooperative scenario. In non cooperative scenario, we do not have restriction on the number of subcarriers used by primary users.

In proposed algorithms, we obtain two necessary conditions for power and subcarrier allocation through the set of Lagrange multipliers and using a Karush-Kuhn-Tucker (KKT) condition. Considering the constraint on the number of primary users' subcarriers, the problem becomes intractable. Thus, to solve it, we first disregard this condition and solve a problem then we choose the best  $K_{th}$  subcarriers and allocate power to them according two different methods. Simulation results are provided to validate our proposed scheme.

The rest of the paper is organized as follows. In Section II, the system model is described and the problems for the

given system are formulated. In Section III, the proposed algorithms and condition for optimum solution are presented. Simulation results are provided in Section IV. Finally we conclude in Section V.

# II. SYSTEM MODEL AND PROBLEM FORMULATION

#### A. System model

We consider resource allocation for the Uplink transmission of a single cell OFDMA-based cognitive relaying network in which a Base Station (BS) serves N primary users and M secondary users. Assume that all the channel state information (CSI) can be properly obtained at BS.

It is assumed that there are K OFDM subcarriers in system. Each subcarrier is allocated to only one user and the length of each subcarrier is much less than the coherent bandwidth of channel. Thus, the channel response on each subcarrier is flat. It is assumed that system is time-slotted which time slot duration equal to an OFDM symbol. To synchronous all users, the beginning and the end of each time slot are known by them.

The transmission of primary users is protected by enforcing the secondary users don't use carriers of primary users and send their data in unutilized subcarriers. Moreover secondary users have to exit transmission whenever primary users come back. Secondary users try not to interfere with primary users and also improved the performance of primary users by relaying data of primary users.

We assume that communication of a primary user takes place in two time slots. In first time slot, a primary user sends data to BS in its own subcarriers. In this time slot, secondary users can receive data of the primary users. In second time slots, for each subcarrier secondary user that has highest relay-BS channel gain is chosen to be relay. The relay amplifies primary user data with coefficient A and then sends to BS.

The received signals from subcarriers k of  $n^{th}$  primary user  $(PU_n)$  at BS and  $m^{th}$  cognitive users  $(CR_m)$  in the first time slot are respectively given by:

$$y_{PU_{n},BS}^{k} = \sqrt{P_{n,k}} H_{PU_{n},BS}^{k} x_{PU_{n}}^{k} + Z_{PU_{n},BS}$$

$$y_{PU_{n},CR_{m}}^{k} = \sqrt{P_{n,k}} H_{PU_{n},CR_{m}}^{k} x_{PU_{n}}^{k} + Z_{PU_{n},CR_{m}}.$$
(1)

Where  $P_{n,k}$ ,  $x_{PU_n}^k$  are respectively defined as transmit power and signal of  $PU_n$  on subcarrier k. Generally,  $H_{a,b}^k$  denotes the channel gain between node a and node b at subcarrier k which for different a and b is assumed to be zero-mean, independent, circularly symmetric complex Gaussian random variables.  $H_{a,b}^k$  incorporates the effects of path loss and shadowing.  $Z_{PU_n,BS}$  and  $Z_{PU_n,CR_m}$  are additive white Gaussian noise (AWGN) with variance  $N_0$ .

In the second time slot, the received signal of selected

relay for subcarriers k of  $n^{th}$  primary user  $(r_{n,k})$  is:

$$y_{r_{n,k},BS}^{k} = H_{r_{n,k},BS}^{k} A(y_{PU_{n},r_{n,k}}^{k}) + Z_{r_{n,k},BS} .$$
(2)

The coefficient A is chosen in such ways that transmit power of  $r_{n,k}$  is equal to  $P_{r_{n,k}}$ .

Consequently after Maximal Ratio Combining (MRC) at BS, the received signal-to-noise ratio (SNR) of  $PU_n$  at subcarrier k,  $\gamma_{n,k}$ , can be written as:

$$\gamma_{n,k} = \eta_{CR_m,BS}^k + \frac{\eta_{PU_n,CR_m}^k \eta_{CR_m,BS}^k}{\eta_{PU_n,CR_m}^k + \eta_{CR_m,BS}^k + 1}.$$
 (3)

Where  $\eta_{a,b} = \frac{P_a |H_{a,b}^k|}{N_0}$ , is SNR of link between node *a* 

and node b, where node a as transmitter dedicates power  $P_a$  for transmission data. In the case of independent and identically distributed zero-mean, circularly symmetric complex Gaussian input, the maximum mutual information between  $n^{th}$  primary user and BS at subcarrier k under cooperation is given by [7]:

$$I_{PU_n} = \log_2(1 + \gamma_{n,k}).$$
 (4)

Where  $b = [b_{11}, b_{12}, ..., b_{1K}, b_{21}, ..., b_{NK}]^T$  is a vector that indicates presence of primary users on subcarriers, that is,  $b_{nk} = 1$  if subcarrier k is allocated to  $PU_n$  and  $b_{nk} = 0$  if subcarrier k is not allocated to  $PU_n$ , that is:

$$\sum_{n=1}^{N} \mathbf{b}_{n,k} \le 1, \qquad \forall k \,. \tag{5}$$

# B. Problem formulation

Our objective is to maximize the sum rate for primary users subject to total transmit power of each primary user and the number of subcarriers that used by primary users constraints. The last constraint only applies when the desired rate of primary users is achieved. Thus, if we assumed that in cooperative transmission primary users achieved their desired rate, we can formulate our problem as:

$$\max_{P,b} \sum_{n=1}^{N} \sum_{k=1}^{K} b_{n,k} \log_2(1+\gamma_{n,k})$$
(6)

$$\sum_{n=1}^{N} \sum_{k=1}^{K} b_{n,k} \leq K_{ih} \quad \forall k$$
<sup>(7)</sup>

$$\sum_{k=1}^{k} P_{n,k} \le P_n \tag{8}$$

$$\sum_{n=1}^{N} b_{n,k} \le 1 \tag{9}$$

$$P_{n,k} \ge 0 \tag{10}$$

$$b_{n,k} \ge 0 \tag{11}$$

Where  $K_{th}$  is the maximum number of subcarrier that used by primary users and  $P_n$  denotes the total allowed power of  $n^{th}$  primary user. The optimization variables are primary subcarrier allocation indicator vector, i.e. *b* and primary power allocation vector, i.e. *P*. Equation (7) represents that primary users should allocate minimum subcarriers to secondary users. The number of  $K_{th}$  should be determined in a way that in spite of constraint (7), the performance of primary users improves. We will show that despite this limit, primary users gain advantages of cognitive user cooperation. Constraint (8) corresponds to total power that used by  $n^{th}$  primary user. Inequality (9) follows from assumption that a subcarrier can be allocated to at most one user.

### III. PROPOSED ALGORITHM

An optimal solution to integer programming problem in (7) is computationally complex. To become the optimization problem more tractable, the noise in

PU – relay link, i.e.  $Z_{PU_n,r_{n,k}}$  is neglected, therefore  $\gamma_{n,k}$ 

can be written as:

$$\gamma_{n,k} \approx P_{n,k} \frac{\left| H_{PU_{n,d}}^{k} \right|^{2}}{N_{0}} + P_{r_{n,k}} \frac{\left| H_{r_{n,k},d}^{k} \right|^{2}}{N_{0}}$$
(12)  
$$\gamma_{n,k} \approx a_{n,k} P_{n,k} + c_{n,k}.$$

Despite this approximation the optimal solution is difficult thus; we try to find suboptimal solution for this problem. At first we solve this problem by disregarding condition (7). Then we select the best  $K_{th}$  subcarriers out of K subcarriers. We consider two different power allocations for  $K_{th}$  selected subcarriers. The details will be given later. Considering the approximation in (12), the expression (6)-(11) form a convex optimization problem because the objective function is convex in optimization variables, and all constraints are linier in them. The convex program can be solved using general solution techniques; however we use an approximately optimal solution base on Lagrangian theory [8].

The Lagrangian of the convex program is obtained by introducing Lagrangian multipliers  $\mu \le 0$ ,  $\gamma \le 0$ ,  $\kappa \le 0$ ,  $\nu \le 0$  for constraints (9)-(11) respectively. It should be noted that we assume  $b_{n,k} \in [0,1]$  but it can be shown that the relaxation of  $b_{n,k}$  from 0 or 1 to [0,1] does not change the optimal value. This leads to:

$$L(P,b,\mu,\gamma,\kappa,\nu) = \sum_{n=1}^{N} \sum_{k=1}^{K} b_{n,k} \log_2(1+a_{n,k}P_{n,k}+c_{n,k}) + \sum_{n=1}^{N} \mu_n \left(\sum_{k=1}^{K} P_{n,k} - P_n\right) + \sum_{k=1}^{K} \gamma_k \left(\sum_{n=1}^{N} b_{n,k} - 1\right) - \sum_{n=1}^{N} \sum_{k=1}^{K} \nu_{n,k} P_{n,k} - \sum_{n=1}^{N} \sum_{k=1}^{K} \kappa_{n,k} b_{n,k}.$$
(13)

Where  $a_{n,k}$ ,  $c_{n,k}$  are defined in (12).

From (6)-(11), Karush-Kuhn-Tucker (KKT) optimality conditions are given by:

$$\frac{\partial L(p,b,\mu,\gamma,\kappa,\nu)}{\partial P_{n,k}} = b_{n,k} \times \frac{a_{n,k}}{1 + a_{n,k}P_{n,k} + c_{n,k}} + \mu_n - \nu_{n,k} = 0$$
(14)

$$\frac{\partial L(p,b,\lambda,\mu,\gamma)}{\partial b_{n,k}} = \log_2(1 + a_{n,k}P_{n,k} + c_{n,k})$$
(15)

$$+\gamma_{k} - \kappa_{n,k} = 0$$

$$\mu_{n} \left( \sum_{k}^{K} P_{nk} - P_{n} \right) = 0$$

$$\gamma_k \left( \sum_{n,k}^N b_{n,k} - 1 \right) = 0 \tag{17}$$

(16)

$$P_{n,k}v_{n,k} = 0 \tag{18}$$

$$b_{n,k}\kappa_{n,k} = 0. \tag{19}$$

From (14)-(19) and after some manipulations, we conclude that subcarrier k should be allocated to user i selected by,

$$\max \log_2(1 + a_{x,k} P_{x,k} + c_{x,k}).$$
(20)

The optimal transmit power allocation for subcarrier k of  $n^{th}$  PU using the Karush–Kuhn–Tucker condition is given by:

$$P_{n,k} = \left(\Delta - \frac{1}{\Gamma_{n,k}}\right)^+.$$
 (21)

Where  $\Gamma_{n,k} = a_{n,k} (1 + c_{n,k})^{-1}$  and  $\Delta$  is the water-filling level which makes the total power of primary user *n* equal to power constraint.

After determining transmits power for subcarriers of each primary user by using the water-filling in (21), we allocate subcarriers to appropriate primary user in which maximizes the rate given in (20). Thus, we have:

$$\max_{x} \log_2(1+a_{x,k}\left(\Delta - \frac{1}{\Gamma_{x,k}}\right)^* + c_{x,k}).$$
 (22)

Now, we should consider the constraint (7) in resource allocation. After determining which subcarriers assigned to which primary users, we should select  $K_{th}$  subcarriers out of K ones. To do this, we select  $K_{th}$  subcarriers which have max rate in resource allocation problem without constraint (7). It means that the way we assigned subcarriers to primary users will not change in this stage and we assume the same subcarrier allocation for this resource allocation problem. But, we consider different power allocation because, the sum of power allocations for  $K_{th}$  selected subcarriers not equal to defined power constraint in (8). Thus, to hold the power constraint for  $K_{th}$  selected subcarriers, we should increase power of selected subcarriers that can be done in two different ways. First, we consider the power allocation defined in (21) then increase them with constraint rate  $P_n \times \left(\sum_{k \in T_-} P_{n,k}\right)^{-1}$ , where  $T_n$  is a set of selected

subcarriers for  $n^{th}$  primary user.  $P_n$  is the total allowed power of  $n^{th}$  primary user that define in (8).

Second, we apply water-filling method for selected subcarriers.

# IV. SIMULATION RESULTS

In this section, we present the performance of the proposed algorithms by computer simulations. In our simulation, the number of primary users, the number of cognitive users, and the number of subcarriers are respectively 3, 4, and 64. Channel gain for relay-BS,  $PU_n$ -BS and  $PU_n$ - $r_n$ , are assumed to be zero-mean, independent, circularly symmetric complex Gaussian random variables with variances -10 dB. We assume that the desired rate is the sum rate of primary users achieved in non cooperative scenario.

Fig. 1 shows the sum rate of primary users versus the SNR of relays for non cooperative scenario and cooperative scenario for the first method of power allocation. In non cooperative scenario, whole subcarriers are assigned to primary users. For comparison, we illustrate the cooperative scenario in which whole and partial numbers of subcarriers are assigned to primary users. From Fig.1, when primary users used all subcarriers, cooperative scenario outperforms non cooperative scenario. Even though, in the cooperative scenario in which primary users and have constraint on the number of subcarriers, it still surpasses the cooperative scenario. Against

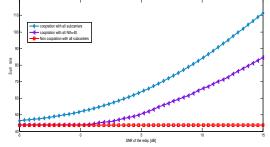


Fig.1. achieve rate for primary users for cooperative and non-cooperative scenario.

Fig. 2 illustrates the sum rate of primary users versus SNR of the relay for different numbers of  $K_{th}$  for the first method of subcarrier selection. As expected, by increasing the number of subcarriers used by primary users ( $K_{th}$ ), the performance of primary users improves. However, the reduction of sum rate of primary users as a result of decreasing  $K_{th}$  can be compensated by the increasing the power of cognitive users dedicated for relaying.

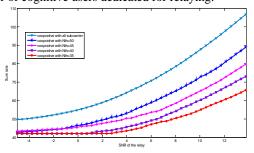


Fig.2. achieve rate of primary users for different numbers of selected subcarriers.

Fig.3 shows sum rate of primary users versus the SNR of cognitive users for relaying for two different methods

of power allocation for selected subcarriers. In this Figure, we assume  $K_{th} = 45$ . From Fig.3, the second method in which power is allocated to selected subcarriers according to water-filling method has better performance, as expected. Although the first method is simpler that the second one, to get better performance, it is reasonable to apply the second method.

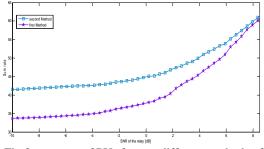


Fig.3. sum rate of PUs for two different methods of power allocation for selected subcarriers.

# V. CONCLUSION

In this paper, we have considered cognitive relay network in which cognitive users act as a relay for primary users. We have investigated the resource allocation for primary users with two constraints. It has been assumed that if primary users achieve their minimum sum rate, they use only limited number of subcarriers. Cognitive user cooperation has been shown to improve the sum rate of primary users. We have considered two different methods of power allocation for selected subcarriers. It has shown that the first method has better performance.

### VI. ACKNOWLEDGMENT

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