On the Outage Probability and BER of MRC and EGC Space-Diversity Reception over Ricean Fading with LMMSE Channel Estimation

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***Abstract*—In this paper, the effect of imperfect channel estimation (ICE) on maximal-ratio combining (MRC) and equal-gain combining (EGC) in receive antenna diversity systems with *M*-level quadrature amplitude modulation (*M*-QAM) are examined. The channel is assumed to be spatially-independent frequency-flat with Ricean fading and AWGN. Average BERs and outage probability of MRC and EGC antenna diversity combining schemes are analyzed via Monte-Carlo simulations versus average received bit SNR per antenna and specified outage threshold γ_{th} respectively for various values of Ricean K-factor, number of antennas L and normalized channel estimation correlation coefficient ρ to show the actual diversity gain with these combining schemes under realistic LMMSE-based channel estimation scenarios for pilot-symbol aided modulation (PSAM).**

***Keywords* — Diversity reception, minimum mean square error channel estimation, quadrature amplitude modulation (QAM), Ricean fading channel.**

I. INTRODUCTION

DİVERSİTY reception is a classical method used in wireless communication systems for combating the deleterious effects of multipath fading [1]. Various techniques can be used for diversity combining such as maximal-ratio combining (MRC) and equal-gain combining (EGC) [2,3]. Many previous performance analyses of coherent diversity systems in literature assume that the receiver has perfect knowledge of the fading channels' gain and/or phase at the receiver. However, this assumption is too idealistic for practical wireless systems. Recently, considerable attention has been paid to examine the performance degradation caused by these estimation errors [4]. Pilot-symbol-assisted channel-estimation (PSA-CE) scheme and estimation based on linear minimum mean-square-error (LMMSE) has long been studied [5]. In addition to the average bit error rate (BER), outage probability denoted by P_{out}, is another standard performance measure of diversity systems operating over fading channels. The probability of outage is defined as the probability of the event that the combined total symbol SNR $\gamma_{s,tot}$ falls below a certain specified threshold γ_{th}:

$$P_{out} = Pr\{\gamma_{s,tot} \leq \gamma_{th}\} \quad (1)$$

Average BER expression for MRC diversity for square M-QAM with pilot-symbol assisted modulation (PSAM) is derived in [5]. In [6], a closed-form expression for the output SNR distribution of the MRC combiner is provided as a function of the correlation between the actual fading values and their estimates. Exact average BERs for M-QAM with MRC and imperfect channel estimation in Ricean fading channels were developed in [7]. In this paper, the outage probability and the average BER performance of M-QAM systems are examined for MRC and EGC diversity-combining receivers operating on i.i.d. Ricean fading channels with pilot-symbol assisted imperfect channel estimation via LMMSE resulting in a Gaussian-error model between the actual and the estimated channel coefficients.

The rest of the paper is organized as follows. The system and the channel model are discussed in Section II. Section III considers the pilot-symbol assisted LMMSE channel estimation with Gaussian-error model for i.i.d. fading channels. Numerical results and discussions via Monte-Carlo simulations under this scheme and model are presented and discussed in Section IV. Section V finally concludes the paper.

II. SYSTEM MODEL

As presented in Fig.1, in the single-user communication system scenario considered, we assume that there are L spatial diversity channels carrying the same information bearing signal. Each channel is modelled as frequency-flat Ricean fading corrupted by additive white Gaussian noise (AWGN) processes. The fading and noise processes among the L spatial channels are assumed to be mutually statistically independent In MRC which is the optimal linear diversity combiner, the individual branches are first co-phased, weighted proportionately to their channel gain and then summed up. This is equivalent to weighting each branch by the complex conjugate of its channel gain, i.e.$h_l = \propto_l exp(-j\emptyset_l)$ [8]. The received discrete signal at the lth antenna is given by:

$$y_l = h_l x + n_l \quad (2)$$

where h_l is the channel coefficient on the lth path, x is the transmitted M-QAM symbol and n_l is the noise sample on the lth path. The combined signal in the noise-free case is given by:

$$y = \sum h_l y_l = x_l \sum \alpha_l^2, \quad (3)$$

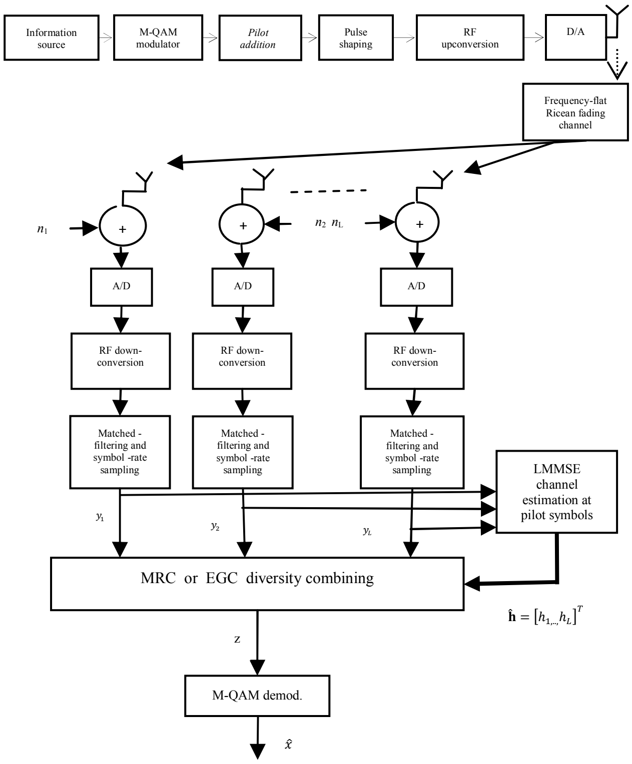

Fig.1 System model of multiple-antenna receive diversity M-QAM system with PSAM and LMMSE channel estimation

The zero-mean noise samples in all branches have equal variance $\sigma_{n,l}^2$ and the total noise power after combining is the sum of the noise powers in each branch weighted by the corresponding gain factors:

$$\sigma_{n,tot}^2 = \sum_{l=1}^{L} \alpha_l^2 \, \sigma_{n,l}^2 \qquad (4)$$

The total average output SNR with MRC is then given by:

$$\bar{\gamma}_s = \sum_{l=1}^{L} \bar{\gamma}_{s,l} = \sum_{l=1}^{L} \frac{\bar{E}_s}{N_0} \qquad (5)$$

that is the sum of the branch average SNRs and $\bar{E}_s$ is the average symbol energy over the M-QAM constellation.

Although suboptimal, equal-gain combining with coherent detection is often an attractive diversity combining technique since it does not require the estimation of the fading amplitudes and hence results in a reduced complexity receiver relative to the optimum MRC scheme [1]. In EGC, the received signals are co-phased in each branch with respect to the phase $\emptyset_l = (l = 1,2..,L)$ of the corresponding desired component $g_l = exp(-j\emptyset_l)$ and then summed up. The complex baseband signal at the output of the EGC receiver can be expressed as:

$$r = \sum_{l=1}^{L} e^{-j\emptyset_l} r_l = (\sum_{l=1}^{L} h_l)x + \sum_{l=1}^{L} e^{-j\emptyset_l} n_l \qquad (6)$$

The total average output SNR with EGC is then given by:

$$\bar{\gamma}_s = \sum_{l=1}^{L} \bar{\gamma}_{s,l} = \sum_{l=1}^{L} \frac{\mu_c^2 \bar{E}_s}{N_0} \qquad (7)$$

III. CHANNEL ESTIMATION VIA PSAM AND LMMSE

In this section, the Gaussian-error model suitable to model estimation error in pilot-symbol aided modulation (PSAM) schemes with LMMSE channel estimation is introduced. The channel state $\mathbf{h}=[h_1,h_2....h_L]^H$ modelling flat Ricean fading channel for each antenna is a proper non-zero mean complex Gaussian random vector with autocorrelation matrix:

$$\mathbf{R_h} = E\{\mathbf{hh}^H\} = \mathbf{I}_{L\times L} \qquad (8)$$

assuming independent channel coefficients of unity path power. The channel noise $n=[n_1,n_2.....n_L]^H$ is also a proper zero-mean complex white Gaussian vector with covariance matrix

$$\mathbf{c_n} = E\{\mathbf{nn}^H\} = \frac{N_0}{2}\mathbf{I}_{L\times L} \qquad (9)$$

We assume that an imperfect channel observation is obtained at the receiver through pilot-symbol aided modulation (PSAM) and LMMSE channel estimation. In a general PSAM scheme as presented in Fig.2, pilot symbols known to the receiver are inserted periodically into the data sequence prior to pulse shaping, and the composite signal is transmitted over the fading channel with AWGN. The pilot symbols maybe written as an F×1 column vector $\mathbf{x}_{ps}$= $[x(i - PF_1 + i_{off}),, x(i - P + i_{off}), x(i +$ ioff,…,xi+PF2−1+iofffT, where F is the total number of pilot symbols employed to estimate the channel coefficients vector. F_1 and F_2, where F_1+F_2=F, are the numbers of pilot symbols on the left and right sides of x(i), respectively, and $i_{off} = (i_{off} = 1,2, \ldots, P-1)$ is the offset of the desired symbol x(i) to the closest pilot symbol on its right side [5].

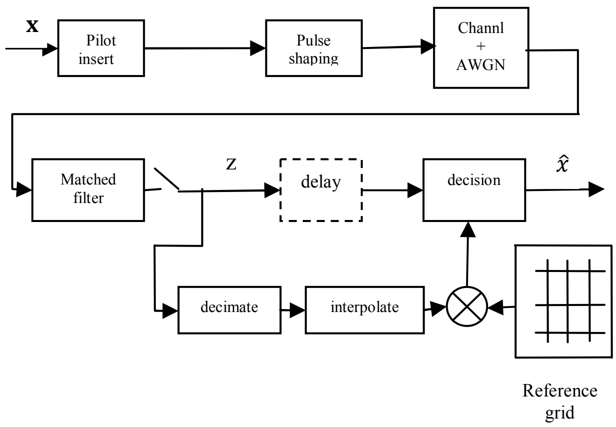

Fig.2 Diagram of a general PSAM scheme.

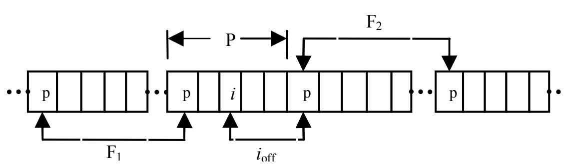

Fig.3 Illustration of PSAM-based channel estimation.

The resulting frame structure is shown in Fig.3 after matched- filtering, the receiver splits the per-symbol samples into two streams. The reference branch decimates the samples to extract those due to the pilot symbols, and interpolates them to form an estimate of the channel state. It then scales and rotates a reference decision grid with the estimate, and feeds the modified decision boundaries to the data branch [9].

After PSAM, maximum-likelihood (ML), linear minimum mean-square error (LMMSE) or decision-feedback (DF) channel estimation schemes can be performed to estimate and periodically update the channel coefficients. Focusing our concentration in this work on LMMSE estimation due to its ease of implementation and low computational complexity, it is sufficient to use single-channel per-antenna LMMSE estimators on each antenna due to the independence of channel coefficients over antennas. Via the sampled signal model after matched-filtering in (2), the cost function to be minimized in LMMSE channel estimation is:

$$C(h_l) = MSE = E\left\{\left|y_l - h_l x_{ps}\right|^2\right\} \qquad (10)$$

That is the mean-squared error (MSE) between the matched-filter output on lth antenna and the pilot-symbol x_{ps} that is known to the receiver weighted by the channel coefficient of the lth antenna. Taking the derivative of MSE with respect to h_l, equating to 0 and solving for h_l yields the following LMMSE estimator for the channel coefficient on lth antenna:

$$\hat{h}_l = \frac{y_l}{x_{ps}} \qquad (11)$$

To evaluate the performance of diversity-combining schemes over fading channels with PSAM and estimators of ML, LMMSE and DF types, Gaussian-error model is widely employed that relates the actual and estimated channel coefficients via an estimation correlation metric and with an additional Gaussian error term [4,5,10].

Under Gaussian-error model, the actual channel coefficients and its estimates are related by [4]:

$$h_l = \hat{h}_l + e_l \qquad (12)$$

Using a diversity-combiner with weight vector $\mathbf{W} = [w_1 \ldots w_L]^T$, where $W_l = \hat{h}_l$ for MRC and $W_l = e^{-j\angle \hat{h}_l}$ for EGC, a combined random variable z that is a sufficient decision-statistic is formed as:

$$z = \boldsymbol{W}^H \boldsymbol{y} \qquad (13)$$

In this work, a Ricean fading environment that is frequency-flat for each antenna is used. In Ricean fading channel model, the propagation paths consist of one strong line-of-sight (LOS) or specular component corresponding to the mean of the Gaussian channel coefficient and many random weaker diffuse components [1].The Ricean K-factor is then defined as the ratio of the power in the specular component to the power in the scattered components. For K=0, the channel exhibits Rayleigh fading, and for K=∞, the channel has no fading corresponding to an AWGN channel. In relation to Gaussian-error model, for the diffuse component, the channel estimation error model for the lth antenna is $h_{f,l} = \hat{h}_{f,l} + e_{f,l}$ where $e_{f,l}$ is the channel estimation error term and is assumed to be independent of $h_{f,l}$. The estimation error term $e_{f,l}$ is zero-mean and follows a complex Gaussian distribution $e_{f,l} \sim CN(0,(1-|\rho|^2)\sigma_c^2)$ where the parameter ρ is the normalized estimation error correlation coefficient between the actual and estimated channel coefficients. Furthermore, the specular LOS component and its estimate follow the same relationship as the diffuse components, i.e. $\mu_{c,l} = \hat{\mu}_{c,l} + e_{\mu,l}$ where $e_{\mu,l}$ is the channel estimation error term for the LOS component and is Gaussian distributed with $e_{\mu,l} \sim CN(0,(1-|\rho|^2)|\mu_c|^2)$. With these relations, for l=1,…..,L $h_l = \hat{\mu}_{c,l} + h_{f,l}$, $e_l = e_{f,l} + e_{\mu,l} \sim CN(0,(1-|\rho|^2)[\rho_c^2 + |\mu_c|^2])$. In this manner, assuming both the actual and estimated channel coefficients to have unity mean-squared value (unity path power), a single correlation coefficient $\boldsymbol{\rho}$ serves as a sufficient indicator of the accuracy of the LMMSE channel estimation [11]. The normalized estimation error correlation coefficient $\rho \in [0,1]$ and $\boldsymbol{\rho}$=1 indicates a system with perfect channel estimation.

IV. NUMERICAL RESULTS AND DISCUSSION

In this section, we present numerical results via Monte-Carlo simulations to present the impact of imperfect channel estimation errors on the performance in conjunction with main system parameters. In order to better illustrate the effect of estimation correlation coefficient$\{\rho_l\}_{l=1}^{L}$, we assumed i.i.d diversity branches, and that the $\{\rho_l\}_{l=1}^{L} = \rho$ are identical for all antennas.

In Fig.4 and Fig.5, we present the outage probability of MRC and EGC receivers with 4-QAM, four antennas and two different values of Ricean K-factor through Monte Carlo simulations for various channel estimation accuracies. Fading and channel estimate realizations are generated as complex Gaussian vectors for the specified estimation accuracy. We observe that the outage probability corresponding to a certain SNR threshold increases as the quality of the channel estimate degrades, as quantified by parameter ρ.

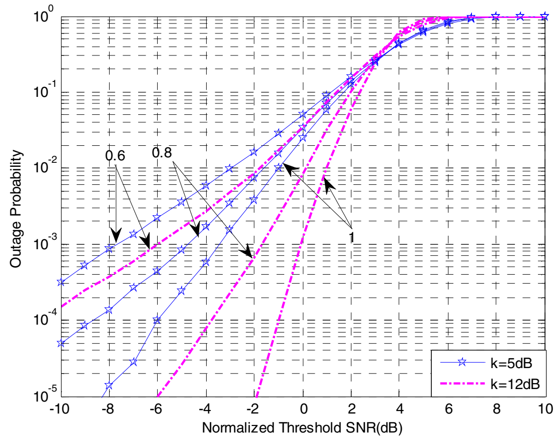

Fig.4. Outage probability of MRC with 4-QAM antennas, L=4 Ricean fading parameter, K=5dB, K=12dB and ICE correlation coefficient ρ= {0.6, 0.8, 1}.

Further comparing Fig.4 and Fig.5 for MRC and EGC respectively, MRC has 2 to 3 dB gain above EGC since EGC is suboptimum with respect to MRC. Moreover, as normalized SNR threshold increases, the detrimental impact of fading severity (lower K-factor) decreases for both MRC and EGC, and for fixed normalized SNR threshold and ICE normalized estimation correlation coefficient ρ, the impact of improved fading severity has less impact on P_{out} for MRC than EGC.

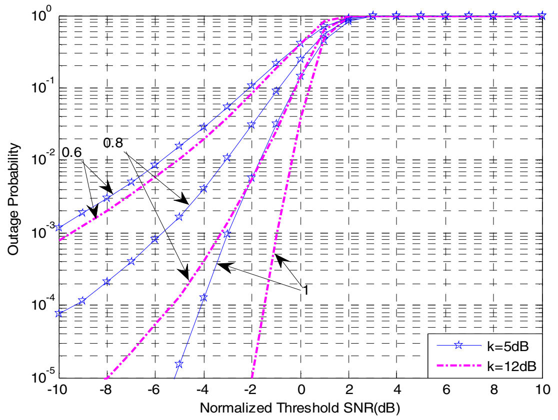

Fig.5. Outage probability of EGC with 4-QAM antennas, L=4 Ricean fading parameter, K=5dB, K=12dB and ICE correlation coefficient ρ= {0.6, 0.8, 1}.

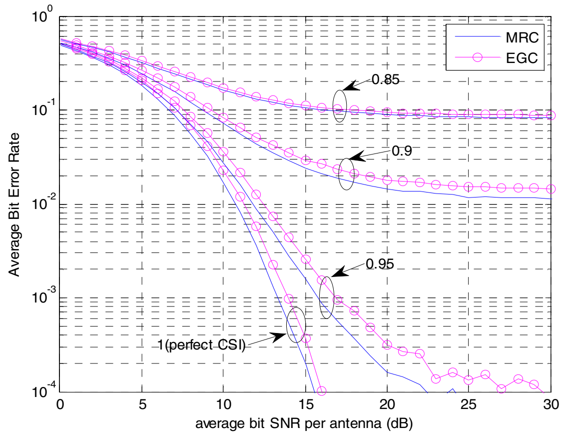

Fig.6. The average BER of MRC and EGC with 16-QAM antennas, L=4 Ricean fading parameter, K=5dB, and ICE correlation coefficient ρ= {0.85,0.9,0.95,1}.

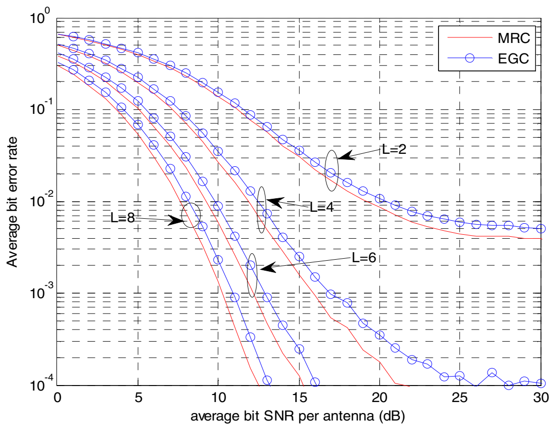

Fig.7. The average BER of MRC and EGC with 16-QAM and ICE(ρ=0.95) with different L antenna number where Ricean parameter, K=5dB.

In Fig.6, we present the effect of varying ICE accuracy when Ricean parameter K=5dB. The performance of MRC and EGC receivers with 16-QAM degrades very rapidly as channel estimation accuracies degrade for ρ decreasing and we observe error-floors in BER curves for all imperfect channel estimation cases at $\rho < 1$.

The average BERs for 16-QAM MRC and EGC receivers with ICE are presented In Fig.7 for different diversity order L, where K=5dB and ρ=0.95. As observed, the average BER performance vastly improves as the number of diversity branches combined increases and error floors due to imperfect channel estimation vanishes after L=6.

V. CONCLUSIONS

In this paper, we investigated the outage probability and average BER of MRC and EGC space-diversity reception over flat Ricean fading channels with PSAM and LMMSE channel estimation. Via Monte-Carlo simulations, we examined the effect of channel estimation inaccuracies represented by the normalized estimation correlation between the actual channel coefficients and their estimates under Gaussian-error model in conjunction with other key system parameters such as number of antennas and Ricean K-factor. The results presented in this paper are expected to provide useful information and guidelines to wireless systems design engineers to exploit the use of diversity combining under realistic imperfect channel estimation scenarios.

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