Transmit Beamforming based on Fairness Switched Sub-Codebook

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Abstract**—In this paper, we consider the technique of limited feedback Transmit Beamforming for MISO systems. In conventional schemes, all vectors of a large codebook, used for the feedback of the quantized channel state information, are broadcasted to all users, in a guard period which is followed by data burst period. *Instead of transmitting a high number of codevectors, the codebook is divided into several sub-codebooks and the broadcast would be based on the switch between them. Accordingly, good performance can be provided while minimizing the required feedback channel rate. An access control is used to provide fairness among users based on proportional fairness constraint. The Beamforming codebook vectors $\{W_i\}_{i=1}^{N}$ ($N = 2^B$) is grouped into n_D sub-codebooks. At each channel use, the transmitter switches to a given sub-codebook adaptively to the users owning the worst case performance $\min_{k \in K}(C_k)$. Simulation results are used to show the performance of the proposed scheme.**

key words- MISO, Beamforming, limited feedback, CSI, switched sub-codebook, score, fairness.

I. INTRODUCTION

In the last few years, Multiuser MISO systems also named as SDMA (Space Division Multiple Access) based on Limited Feedback Transmit Beamforming have received a lot of interests in recent research studies. The goal is to provide high system spectral efficiency [1] while reducing the complexity. Due to the constraint of narrowband of the feedback channel, beamforming transmission on the broadcast channel with limited feedback has been widely studied in the literature as [2], [3] and [4] and references therein.

Moreover, transmit beamforming is provided in the literature as a more practical design that ameliorate the performance of SDMA [5] and [6]. Where each user selects the correspondent beamformer from the Beamforming codebook. Therefore, we found many techniques to formulate the Beamforming codebook (BC) vectors, for example: random orthonormal beamforming codebook as proposed in [7], [8] and [9] when vectors are generated with zero correlation or transmit beamforming based on grassmannian line packing as described in [10] and [11] when the term of codebook with non zero correlation.

This SDMA design can be combined with multiuser scheduling [5] and [6]. Therefore BS uses an algorithm to select the best pair index of user-CB vector that increases the system capacity such as: Max-rate [4] and [12] that is based on maximizing system capacity by allocating each spatial channel to the user that experiences peak level signal to interference plus noise ratio (SINR) or sub-optimal algorithms as proposed in [13] where a selection is based on the best pair of user-beam vectors. In [14], an algorithm is proposed, known as semi-orthogonal user selection scheme. This algorithm is based on upper bounded techniques where the value of the $SINR$ and the value of the error quantization are compared to predefined thresholds which are defined in [2], [3] and [4].

Up to the moment, the generated CB vectors are used randomly to estimate the CSI or transmitted to all users and then select the best pair CB vector-user. After investigating these studies, we thought to meet the performance of these systems and to reduce the complexity by introducing a scheme based on a score that measure frequency of access by using vectors in sub-codebook [12].

In this paper, we propose a scheme based on fairness switched BF codebooks applied to the Max-Rate scheme with limited feedback. This study is the extend of [12] when we investigate and improve the studied technique by introducing the proportional fair principle to switch the specific Sub-CB. Whereby, each user has her own rate β_k. For the goal of providing fairness among users and according to [12], at each time slot, the selected sub-CB vectors is that the most frequently used by the user that has the worst term. This term is expressed as $(\min_{k \in K}\left(\frac{C_k}{\alpha_k}\right))$ where α_k is component that describe β_k. Then, this worst case user would receive more opportunity to access to the channel and give fairness among users. This is still true while slow fading is experienced by user's channels. Then, in this paper, the main goal of the proposal is based on using the benefit of [12] and adding the proportional fairness principle to have a fairness switched sub-codebook.

The remainder of this paper is organized as follows. Section II describes the system and channel models. In Section III, we present an overview of the design of scheme studied in [12] and introduce the proposal fairness idea. In section VI, we analyze the system capacity of the proposed scheme. In section V, we present a selection of simulation results.

II. SYSTEM MODEL

We consider a MISO system with M_t antennas at the base station (BS) and K users when each is equipped with one receive antenna. Each user has her own rate β_k. It is assumed that slow power control is employed

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to equally share the total transmitted power P_t on all transmit antennas at the BS. User symbols are loaded on transmit antennas using Beamforming, i.e the BS assigns a Beamforming vector to each of up to M_t selected active users.

The Beamforming vectors $\{w_i\}_{i=1}^{\mathrm{M_t}}$ are obtained using a generated orthogonal unitary beamforming vectors as defined in [7], [8] and [9] or using grassmannian line packing codebook as described in [2], [3], [4] and [12]. To solve the problem of limited resources allocated to the feedback channel, users estimate their CSI and feed back them in a quantized form on B bits to the BS through an uplink limited capacity feedback (LCFB) channel. We denote by $N = 2^B$ the number of CB vectors which is defined by $CB = \{W_1, W_2, .., W_N\}$. At each time slot, the number of components that would be used in optimal side of the transmission is equal to the number of transmit antennas M_t.

It is assumed that transmit signals experience path loss, log-normal shadow fading, and multi-path fading. The CSI is measured by the vectors which represent the short term fading CSI on all branches from the BS to the k^{th} user assumed to be constant during a time slot. According to the slow power control, (i) each entry of the vector h_k is an independent and identically distributed complex Gaussian random variable $CN(0;1)$ representing the short term fading; (ii) the CSI experiences flat fading during each time slot, and varies independently over time slots.

We denote by $h_k(t)$ the $M_t \times 1$ channel vectors, $W_i(t)$ the $M_t \times 1$ CB, $s(t)$ the $M_t \times 1$ transmitted symbol, $n_k(t)$ the additive white Gaussian noise (AWGN) vector with distribution $CN(0; N_0/2)$ for each element, and $y_k(t)$the received signal.

Then, the received signal for the considered multi-user MISO system in the time slot t is represented by

$$y_k(t) = \sqrt{\frac{P_t}{M_t}} \sum_{i=1}^{M_t} h_k^H(t) W_i(t) s_i(t) + n_k(t),\ k = 1...K \quad (1)$$

According to equation 1, the received signal for user k when using the W_i CB vector can be:

$$y_{k,i} = \sqrt{\frac{P_t}{M_t}} h_k^H W_i s_i + \sqrt{\frac{P_t}{M_t}} \sum_{j=1..M_t, j\neq i} h_k^H W_j s_j + n_k \quad (2)$$

Hence, the corresponding expression of the signal to interference plus noise ratio $SINR$ for the k^{th} user and the i^{th} CB vector is expressed as follows:

$$SINR_{k,i} = \frac{\left|h_k^H W_i\right|^2}{\sum\limits_{j=1..M_t, j\neq i} \left|h_k^H W_j\right|^2 + \frac{M_t}{P_t}} \quad (3)$$

III. THE FAIRNESS SWITCHED SUB-CODEBOOK SCHEME

IV. THE FAIRNESS SWITCHED SUB-CODEBOOK SCHEME

According to [12], specifically to the switched sub-codebook technique that is give a good performance in the studied systems, we are motivated to investigate and improve this technique. Therefore, we thought to introduce an access control based on the proportional fairness (PF) principle in order to guarantee fairness access channel among users. Then, we meet the benefits of [12] and the PF principle to minimize complexity, respect narrowband constraint and provide fairness among users to access to channel.

In this section, we describe the main steps of our proposal scheme.

- CSI Quantization.
- Design of Fairness Switched Sub-Codebook
- Max-rate Scheduling.

In the following, we give a detailed description of these steps.

A. CSI Quantization

Due to constraint of the bandwidth of feedback channel, each user just feeds back its maximum $SINR$ quantized in B bits and the index of the corresponding codebook vector. In [2] and [3], the random vector quantization (RVQ) method is applied for quantizing the CSI. The CB vectors $\{W_j\}_{j=1}^{\mathrm{M_t}}$ are obtained using a generated orthogonal unitary beamforming vectors as defined in [7], [8] and [9] or using grassmannian line packing codebook as described in [2], [3], [4] and [12].

At each time slot, each user identified by k select his 'best' vector from the CB. For that, a quantization CB vector is selected as:

$$i_s = \arg \max_{\{W_j\}_{j=1}^{M_t}} \left|h_k^H * W_j\right| \quad (4)$$

The quantization error is expressed as:

$$\delta = \sin^2(\angle(W_{i_s}, H)) \quad (5)$$

In the literature, it is often assumed for simplicity that the feedback is without errors. Then, the quantization error δ should be converges to zero.

B. Design of Fairness Switched Sub-Codebook

In [12], the main idea is firstly based on dividing the CB of $N = 2^B$ vectors into n_D sub-codebooks. Next, for each user n_D scores are processed from the frequency of access by using each sub-codebook. When, each user has a score on each sub-codebook vector, initialized to zero and incremented after each access using this sub-codebook. In [2], [3] and [4], the N components of the CB vectors are broadcasted to all users at each time slot. This number is reduced in [12] to N/n_D components of the CB that would be broadcasted to all users. Then, we minimize the complexity and respect the constraint of the narrowband of the feedback channel.

Now, we can suppose that each user has her own rate β_k. Then, we will be concentrated on defining how to switch to a given sub-codebook at each time slot to increase the system capacity and give equal opportunity among users to the channel accesses. The switching is based by investigating user scores based on the historic use of each sub-CB at a number of previous time slots and access control based on proportional fairness constraint. This would provide fairness among users.

1) Fairness Criteria: In [12], the idea to give fairness is derived and the selected score is that has the user with the worst capacity ($\min_{k\in K}(C_k)$). If we apply this, we can assume that the number of users to have the worst capacity can be large. Then, the selected user is chosen randomly and the same user can be chosen many times.

Therefore, in our proposal design, the most obvious goal is to give fairness among users. And accordingly, we thought to an idea in basis on max-min schedulers introduced in previous work such as proportional fairness in [7] when all users have the equal chance to be served.

First, we let that each user k has her own rate β_k. second, we propose in our work to select the index of the Sub-codebook that has the minimum of the user's capacity's divided by the proportional component α_k when α_k is expressed in the following sub section in (6).

Then, we can assume that to select user with using $\min_{k\in K}\left(\frac{C_k}{\alpha_k}\right)$ should be give most chance to users who can be served at the following time slot. Accordingly, the probability to choose the same user many times is minimized and this probability converges to zero.

Moreover and according to the most aim of our proposal, we can talk about the index of Jain for fairness defined in [15] and expressed as follows

$$J = \frac{\left(\sum_{k=1}^{K} C_k(t)\right)^2}{K\sum_{k=1}^{K}\left(C_k(t)\right)^2} \tag{6}$$

Where $C_k(t)$ is the system capacity of k^{th} user and K is the total number of users. We are going to present and to discuss this term in the simulation results to validate our scheme and their results.

2) Fairness switching algorithm: The fairness switching algorithm is described as follows:

- Initialization:
  - Let β_k the rate of user k.
  - Let the proportional components α_k as

$$\alpha_k = \beta_k \Big/ \sum_{i=1}^{K}\beta_i; \tag{7}$$

  - Let $Score = zeros(K, n_D)$ is the initialized matrix that describe the channel access.
- The $Score$ matrix is updated at each time slot and should be used in the following time slot:

$$Score(k_s, i_s) = Score(k_s, i_s) + 1; \tag{8}$$

Where k_s is the index of each user among the M_t served users and i_s is the index of each CB vector among the M_t selected CB vectors of the fairness switched sub-codebook satisfying equation (4).

- At each time slot, we search the index of the user that has the worst of the capacity divided by α_k to give fairness among users

$$k^* = \min_{k\in K}\left(\frac{C_k}{\alpha_k}\right); \tag{9}$$

- After, we search the index of the fairness switched Sub-CB vector that satisfies the following expression

$$i^* = \max_i\left(Score\left(k^*, i\right)\right) \tag{10}$$

- Finally, we update the matrix of Scores and compute the system capacity.

The most obvious benefit is to take consideration of the historic use of the CB vectors that will be represented by a score based on the user access frequency [12]. Moreover, the use of an access control based on proportional fairness constraint provide the fairness access channel among users.

C. Max-rate Scheduling

To maximize the system capacity, the technique of scheduling to share resources among active users is studied and applied.

In this section, we describe the main idea of the Max-rate scheduling to select users. Accordingly, the selected M_t users to be served at each time slot experiences peak level signal to interference plus noise ratio (SINR) expressed in (3). This can be expressed as:

$$k_s = \arg\max_k(SINR(k, i_s)); \tag{11}$$

Where i_s is the index of the Fairness switched Sub-Codebook at the correspondent time slot. We use the Max-rate scheduling because it gives the optimal performance and the aim is to investigate the resources with the most efficiently.

V. CAPACITY ANALYSIS

For systems with Limited Feedback, the use of CB vectors to quantify the CSI and the scheduler to select users at the BS increase the sum of system capacity but the quantization error in the CSI feedback decreases it.

According to the fairness switched Sub-codebook design presented in the sub section B.2, the supportable system capacity $C_{k_s\in M_t}(t)$ obtained by introducing the proposal scheme is expressed as

$$C_{k_s}(t) = \log_2\left(1 + \max_k(SINR_k(t))\right) \tag{12}$$

Where $SINR_k(t)$ is the $SINR$ at the receiver of the channel from the pair n_D^{th} fairness switched Sub-codebook vectors and the k_s^{th} users when $k_s \in M_t$ selected users. Then, The instantaneous supportable capacity of the M_t scheduled users , $C(t)$ may be expressed as

$$C(t) = \sum_{i=1}^{M_t} C_{k_s}(t) \tag{13}$$

VI. SIMULATION RESULTS

In this section, numerical results are carried out to show the performance of our proposal. The performance of the Max-rate scheme with limited feedback is evaluated using the fairness switched sub-codebook design for grassmannian CB vectors [10] and [11] in terms of system capacity. These results are compared to the switched Sub-CB scheme studied in [12].

Figure 1 shows the performance of these schemes when the number of sub-codebook n_D is equal to 2 with a time moving window T is of 200 and feedback bits B is of 3.

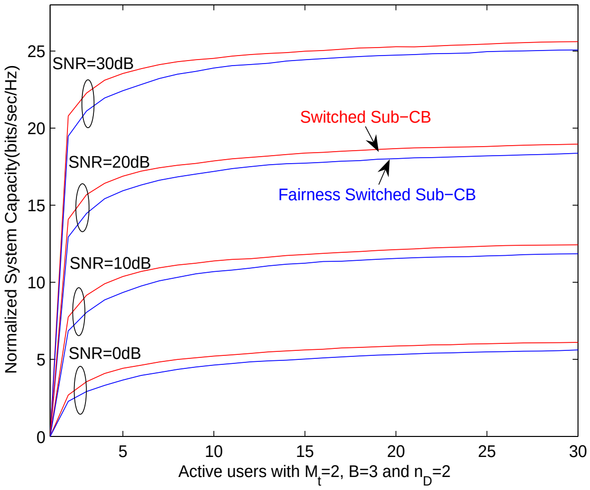

Fig. 1. System capacity of the proposed scheme vs number of active users with $M_t = 2$, $T = 1000$ and $B = 3$

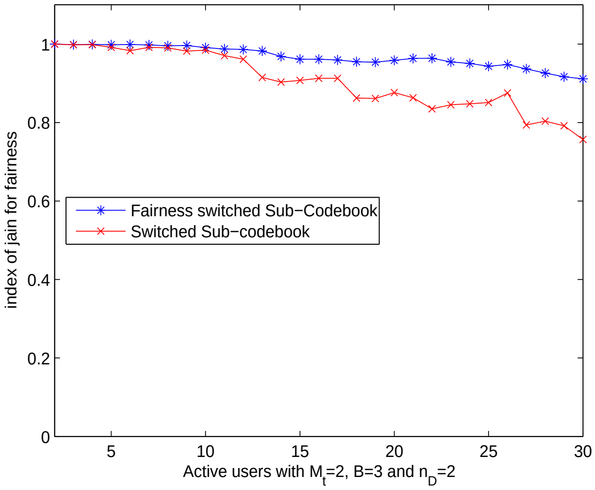

Fig. 2. The index of jain for fairness with $M_t = 2$, $T = 1000$, $K = 30$ and $B = 3$

Figure 1 shows the simulation results of the system capacity of the fairness switched sub-codebook and switched sub-codebook for Max-rate scheme with LF versus the number of active users with different values of average SNR when the number of active users K used for these simulations varies from 1 to 30.

In this figure, we note that the system capacity using grassmannian CB and applying the fairness switched sub-codebook and switched sub-codebook for Max-rate scheme with LF is nearly to saturate for $K > 5$ for different values of SNR. The system capacity increases linearly with the number of users K. We note that the system capacity is nearly independent of K. These simulation results which are carried out for different schemes, illustrate that the results given by fairness switched or switched Max-rate are in good concordance. To confirm this, some simulations are carried out for different average SNR values. In addition to that, the system capacity grows as the average SNR increases.

Figure 2 plots the index of jain for fairness of these two schemes versus the number of active users K when the value of $SNR = 20db$. This Figure shows the fairness degree of the capacity in fairness switched sub-codebook and switched sub-codebook schemes. We can conclude that the fairness switched sub-codebook and switched sub-codebook provide respectively a quasi optimal ($\cong 1$) and near optimal fair degree. This is explained by the number of users that have the same worst capacity can increase with the number of users.

According to these results, we can validate the Design of the fairness switched Sub-Codebook applied to Max-Rate beamforming scheme with limited feedback for a lot of number of users.

VII. CONCLUSION

The system capacity of limited feedback using grassmannian CB vectors and applying the fairness switched sub-codebook or applying the switched sub-codebook for Max-rate scheme has been analyzed in this paper. The transmit antenna are assigned to different up to M_t users at each time slot in term to increase system capacity.

Simulations have been carried out to validate the obtained performance of our proposal scheme. According to these results, we can conclude that the fairness switched sub-codebook for Max-rate give fairness among users for a lot of number of users and a good performance. Moreover, this can be reduce the complexity of generating the CB vectors as switched Sub-codebook scheme.

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