From Physical Mobile Channel to FIR Channel with Stationary, Ergodic Coefficients

Venceslav Kafedziski

***Abstract*—We develop a discrete time Finite Impulse Response (FIR) channel model of a wireless mobile channel. We start with a physical ray model with a number of distinct scatterer clusters. We assume that the positions of scatterer clusters are fixed, but the contribution of each cluster is a sum of contributions of a large number of elementary scatterers. We show that under certain assumptions, the signal obtained as a result of each cluster is a complex Gaussian, stationary and ergodic random process. We then derive an approximate FIR discrete time channel impulse response for the time-varying channel impulse response, which includes the transmit and receive filters. We show that its coefficients vary according to a stationary and ergodic proper complex Gaussian vector random process.**

***Index Terms*—Physical mobile channel, WSSUS channel, proper complex process, FIR channel, stationarity, ergodicity**

I. Introduction

The typical approach to modeling and simulation of a multipath fading channel is by generating path delays from a given probability distribution, (e.g. exponential), and then filtering the complex path amplitudes so that the resulting Doppler power spectrum has the proper shape ([1]). In [1] orthogonalization of the channel impulse response including the transmitting filter, with respect to the channel delay is performed. An alternative Monte Carlo approach is to shape the spectrum by generating a large number of complex exponentials with (Doppler) frequencies drawn from a probability distribution proportional to the Doppler power spectrum ([2]).

An original approach to the channel modeling and simulation based on physical grounds is described in [3]. Doppler shifts depend on the angles between the incoming waves obtained by scattering and the velocity vector. A random spatial distribution of scatterers is assumed. Each scatterer is modeled as a collection of elementary scatterers with equal reflected power; the elementary scatterers are distributed over a narrow range of angles.

We follow [3] to obtain the physical ray model. We assume that the positions of scatterer clusters are given, and that the spatial distribution of elementary scatterers within a cluster is defined by the angle distribution of the incoming waves. We use central limit theorem to prove that the resulting signal of a single cluster is a proper complex Gaussian process.

We then derive a discrete time model for the time-varying channel impulse response, which includes transmit and receive filters, using symbol rate sampling, and show that this model is equal to the one obtained by the best mean square approximation of transmit filter - channel impulse response combination.

Venceslav Kafedziski, Faculty of Electrical Engineering and Information Technologies, University Cyril and Methodius, Skopje, Republic of Macedonia (phone: +38923099120; fax: +38923064262; email: kafedzi@feit.ukim.edu.mk).

The main contribution of this work is to establish that given certain assumption on the distribution of the angles of arrival from the elementary scatterers in the clusters, the complex amplitudes of signals that arrive at the receiver at different delays form a zero mean stationary ergodic proper complex Gaussian vector process in time. This is a starting point to show that the coefficients in the FIR channel model vary according to a stationary ergodic proper complex Gaussian vector process. Ergodicity is important assumption to obtain capacity expressions and to use adaptive modulation techniques on fading channels ([4], [5]). Although FIR channel models similar to the one derived here are well known, usually, stationarity and ergodicity of the time varying FIR coefficients are tacitly assumed, with no deeper analysis or justification.

In Section II we define our physical channel model. We show that our physical model is a Wide Sense Stationary Uncorrelated Scaterring (WSSUS) channel model. We prove that under certain assumption on the angle distribution of elementary scatterers within a cluster, the processes due to the scatterer clusters are ergodic. In Section III we obtain the FIR channel model and prove that FIR coefficients vary according to stationary, ergodic proper complex Gaussian vector process.

II. Physical Mobile Channel Model

One characteristic of the mobile channel as a multipath medium is the time spread introduced in the signal transmitted through such channel. A second characteristic is time variation of channel impulse response as a result of vehicle movement.

In the case when we have discrete paths, the time varying channel impulse response is ([6]):

$$c(\tau, t) = \sum_m a_m(t) e^{-i[2\pi f_c \tau_m(t) + \phi_m(t)]} \delta[\tau - \tau_m(t)] \quad (1)$$

where $a_m(t)$ is the attenuation factor $\tau_m(t)$ is the propagation delay and $\phi_m(t)$ is the retarded phase of the signal received on the mth path.

For times over which the scatterer geometry does not change much due to vehicle movement, we can assume that $a_m(t) \approx a_m$, $\phi_m(t) \approx \phi_m$. Similarly, we can assume $\tau_m(t) \approx \tau_m$ in $\delta[\tau - \tau_m(t)]$, but, in the exponent, we have to keep two terms in the power series for $\tau_m(t)$, i.e. $\tau_m(t) \approx \tau_m + \dot{\tau}_m$. The impulse response is simplified to

$$c(\tau, t) = \sum_m a_m e^{i(2\pi\lambda_m t + \psi_m)} \delta(\tau - \tau_m) = \sum_m c_m(t) \delta(\tau - \tau_m) \quad (2)$$

where $\lambda_m = -f_c \dot{\tau}_m$ is the Doppler frequency shift and $\psi_m = -[2\pi f_c \tau_m + \phi_m]$ is the total phase factor.

Frequently, scattering takes place over a large area of rough (relative to the wavelength) or irregular surfaces such as vegetation or objects close to the antennas. Such scattering is not specular but diffuse, and a cluster of diffusely scattered rays is composed of a multitude of individual rays which exhibit less directivity and no phase coherence. In some cases of interest, all incoherent rays of a cluster have approximately equal propagation delays $\tau_{m,n} = \tau_m + \Delta\tau_{m,n}$, and approximately equal Doppler shifts, $\lambda_{m,n} = \lambda_m + \Delta\lambda_{m,n}$, for $n = 0, \dots, N_m - 1$ ([3]). The complex amplitude of the signal scattered from this cluster is:

$$A_m(t) = \sum_{n=0}^{N_m-1} c_{m,n}(t) = \sum_{n=0}^{N_m-1} a_{m,n} e^{i\psi_{m,n}} e^{i2\pi\lambda_{m,n}t} \quad (3)$$

We assume uniformly distributed phases $\psi_{m,n}$ in $[0, 2\pi]$. If there exist M such clusters with distinct delays τ_m, the channel impulse response becomes

$$c(\tau, t) = \sum_{m=0}^{M-1} A_m(t)\delta(\tau - \tau_m) \quad (4)$$

where we assume that the complex lowpass signal experiences effectively the same delay τ_m for all elementary scatterers with delays $\tau_{m,n}$. This model, with M fixed and relatively small (on the order of 10-20) and all N_m for $m = 0, \dots M-1$ very large will be the one of our interest.

We first characterize random processes $A_m(t)$.

Theorem 2.1: Let the channel model described by (4) and (3) be given. Fix $m \in \{0, \dots, M-1\}$, and denote the number of scatterers in the mth cluster with N. Assume that the amplitudes $a_{m,n}$, $n = 0, \dots, N-1$ of the elementary scatterers of the m-th cluster can be represented as

$$a_{m,n} = \frac{a'_{m,n}}{\sqrt{N}} \quad (5)$$

where $a'_{m,n}$, $n = 0, \dots, N-1$, are independent identically distributed (iid) zero mean random variables with variances $\mathbb{E}[a'_{m,n}]^2 < \infty$. Assume that $\psi_{m,n}$, $n = 0, \dots, N-1$, are uniformly distributed on $[0, 2\pi]$ and independent of $a_{m,n}$, and that $\lambda_{m,n}$, $n = 0, \dots, N-1$, are random variables, independent of $\psi_{m,n}$ and $a_{m,n}$. When the number of elementary scatterers $N \to \infty$, $A_m(t)$ is a zero mean, stationary, proper, complex Gaussian random process.

Proof: Since $A_m(t)$ is complex, we can write $A_m(t) = x(t) + iy(t)$. Consider

$$\begin{aligned} \mathbf{u}_l &= [x(t_1), x(t_2), \dots, x(t_l), y(t_1), y(t_2), \dots y(t_l)]' \\ &= [x_1, x_2, \dots, x_l, y_1, y_2, \dots, y_l]' \end{aligned} \quad (6)$$

From (3), $x_i = Re[z_i]$ and $y_i = Im[z_i]$ where

$$z_i = \sum_{n=0}^{N-1} \frac{a'_{m,n}}{\sqrt{N}} e^{i(2\pi\lambda_{m,n}t_i + \psi_{m,n})} \quad (7)$$

for $i = 1, \dots, l$. It is trivial to show that $\mathbb{E}[x_i] = \mathbb{E}[y_i] = 0$. Therefore, $A_m(t)$ is a zero mean complex process. We first show that the marginal distribution of $A_m(t)$ is Gaussian. To use the central limit theorem we need to show that the variances of $a'_{m,n}\cos(2\pi\lambda_{m,n}t + \psi_{m,n})$ and $a'_{m,n}\sin(2\pi\lambda_{m,n}t + \psi_{m,n})$ are finite. From [7], p.224,

$$\mathbb{E}[(a'_{m,n}\cos(2\pi\lambda_{m,n}t + \psi_{m,n}))^2] = \frac{\mathbb{E}[a'^2_{m,n}]}{2} < \infty \quad (8)$$

$$\mathbb{E}[(a'_{m,n}\sin(2\pi\lambda_{m,n}t + \psi_{m,n}))^2] = \frac{\mathbb{E}[a'^2_{m,n}]}{2} < \infty \quad (9)$$

We show that when $N \to \infty$, $\mathbf{u}_l$ has a joint Gaussian distribution for any finite l. We use the definition ([7] p.186):

Definition 2.1: The random variables $x_1, \dots, x_n$ are jointly Gaussian, if and only if the sum

$$a_1x_1 + \cdots + a_nx_n \quad (10)$$

is a Gaussian random variable for any choice of $a_1, \dots, a_n$. It would be tempting to assume that a linear combination of Gaussian random variables is Gaussian, but this is not true in general. This is true if the components are independent. In our case we form the linear combination

$$\xi = \omega_1x_1 + \omega_2x_2 + \cdots \omega_lx_l + \omega_{l+1}y_1 + \omega_{l+2}y_2 + \cdots + \omega_{2l}y_l \quad (11)$$

where $\omega_1, \omega_2, \dots, \omega_{2l}$ are arbitrary constant weights. According to the central limit theorem, when $N \to \infty$, ξ assumes a Gaussian distribution. By Definition 2.1 $\mathbf{u}_l$ is jointly Gaussian for any finite l. Therefore, $A_m(t)$ is a complex Gaussian process. Note that for any Δt

$$\mathbb{E}[A_m(t+\Delta t)A_m(t)] = \lim_{N\to\infty} \sum_{j=0}^{N-1}\sum_{k=0}^{N-1} \mathbb{E}[a_{m,j}a_{m,k}]$$
$$\mathbb{E}[e^{i(2\pi\lambda_{m,j}(t+\Delta t)+\psi_{m,j}+2\pi\lambda_{m,k}t+\psi_{m,k})}] = 0$$

since $\psi_{m,j}$ and $\psi_{m,k}$ are uniformly distributed in $[0, 2\pi]$. Thus, $A_m(t)$ is a proper complex zero mean Gaussian process. Its statistics can be described in terms of its autocorrelation function $R_m(t, t+\Delta t) = \mathbb{E}[A_m(t+\Delta t)A_m^*(t)]$ only:

$$R_m(t, t+\Delta t) = \lim_{N\to\infty} \sum_{j=0}^{N-1} \mathbb{E}[a_{m,j}^2 e^{i2\pi\lambda_{m,j}\Delta t}] \quad (12)$$

Thus, $A_m(t)$ is a stationary process, since it is Gaussian. ∎

Assume that the mobile is moving with velocity vector $\mathbf{v}$. We set $v = |\mathbf{v}|$. Denote the incident angle between the n-th partial wave $a_{m,n}e^{i(2\pi\lambda_{m,n}t+\psi_{m,n})}$ in the m-th scatterer and the velocity vector $\mathbf{v}$ by $\alpha_{m,n}$. The elementary Doppler shifts are $\lambda_{m,n} = \lambda_{max}\cos\alpha_{m,n}$, where $\lambda_{max} = f_c\frac{v}{c}$ is the maximum Doppler frequency shift for a given velocity v and carrier frequency f_c. Assuming that the amplitudes and angles of the elementary waves are independent, and that all the amplitudes $a_{m,j}$ for $j = 0, \dots, N-1$ are identically distributed, and all the angles $\alpha_{m,j}$ are identically distributed, we obtain

$$\begin{aligned} R_m(\Delta t) &= \lim_{N\to\infty} \mathbb{E}[a_m^2] \sum_{j=0}^{N-1} \mathbb{E}[e^{i2\pi\lambda_{max}\Delta t\cos\alpha_{m,j}}] \\ &= \mathbb{E}[a'^2_m] \int_{\alpha'}^{\alpha''} d\alpha_m p(\alpha_m) e^{i2\pi\lambda_{max}\Delta t\cos\alpha_m} \end{aligned}$$

where $p(\alpha_m)$ denotes the probability density function of $\alpha_{m,j}$ for $j = 0, \dots, N-1$ and $[\alpha', \alpha''] \subseteq [0, 2\pi]$. The average power in the process $A_m(t)$ is $\rho_m = R_m(0) = \mathbb{E}[a'^2_m] < \infty$.

Due to distinct clusters being distant from each other, it is fair to assume that all $A_m(t)$, $m = 0, \dots, M-1$ are independent. Then, the cross-power density is

$$R(\tau,\Delta t)=\mathbb{E}[c(\tau,t+\Delta t)c^*(\tau,t)]=\sum_{m=0}^{M-1}R_m(\Delta t)\delta(\tau-\tau_m) \tag{13}$$

and the model used here to describe the physical mobile channel is a WSSUS channel model ([6]).

In Clarke-Jakes channel model $p(\alpha_m)$ is assumed uniformly distributed in $[0,2\pi]$, i.e. $p(\alpha_m)=1/(2\pi)$, which corresponds to an isotropic incident field. In [8] we derived scattering functions without the assumption of an isotropic incident field. For a uniform distribution of scatterers in an ellipse with the transmitter and receiver in its foci, in [8] we obtained a closed form expression for the scattering function.

Theorem 2.2: Assume that $p(\alpha_m)$ is nonzero on the interval $[\alpha_1,\alpha_2]\subseteq[0,2\pi]$, and is a bounded function (i.e. satisfies the condition $p(\alpha_m)<C$ for any $\alpha\in[\alpha_1,\alpha_2]$, where C is a finite constant). Then the process $A_m(t)$ is ergodic.

Proof: Sufficient condition for a zero mean stationary Gaussian process to be ergodic, is that $\lim_{|\Delta t|\to\infty}R(\Delta t)=0$. Here

$$R(\Delta t)=\lim_{|\Delta t|\to\infty}\rho\int_{\alpha_1}^{\alpha_2}d\alpha p(\alpha)e^{i2\pi\lambda_{max}\Delta t\cos\alpha} \tag{14}$$

By a change of variables $\lambda=\lambda_{max}\cos\alpha$, we obtain

$$\rho\int_{\lambda_1}^{\lambda_2}\frac{s(\lambda)}{\sqrt{\lambda_{max}^2-\lambda^2}}e^{i2\pi\lambda\Delta t}d\lambda=\rho\int_{\lambda_1}^{\lambda_2}X(\lambda)e^{i2\pi\lambda\Delta t}d\lambda \tag{15}$$

where $s(\lambda)$ is bounded since $p(\alpha)$ is (i.e. there exists a finite constant C_1 such that $s(\lambda)\le C_1$ for all $\lambda\in[\lambda_1,\lambda_2]$). From Fourier theory it follows that if $x(t)$ and $X(\lambda)$ are a Fourier transform pair, then $\int|X(\lambda)|d\lambda<\infty$ implies that $\lim_{|t|\to\infty}x(t)=0$. We have $R(\Delta t)=\rho x(\Delta t)$. Our goal is to prove that $x(\Delta t)\to 0$ as $|\Delta t|\to\infty$. It thus suffices to show that $\int|X(\lambda)|d\lambda<\infty$. Denote $X_1(\lambda)=C_1/\sqrt{\lambda_{max}^2-\lambda^2}$. Note that $\int_{\lambda_1}^{\lambda_2}|X_1(\lambda)|d\lambda\le\int_{-\lambda_{\max}}^{\lambda_{\max}}|X_1(\lambda)|d\lambda=\pi C_1<\infty$. From $X(\lambda)\le X_1(\lambda)$ and X, X_1 nonnegative, it follows that $\int_{\lambda_1}^{\lambda_2}|X(\lambda)|d\lambda\le\infty$. Thus, $R(\Delta t)\to 0$ as $|\Delta t|\to\infty$. ∎

All the previous results can be extended.

Theorem 2.3: $[A_0(t),\dots,A_{M-1}(t)]$ is a zero mean stationary ergodic complex proper Gaussian vector process.

Proof: We showed that $A_m(t)$ are zero mean complex Gaussian processes. By assumption all $A_m(t)$ for $m=0,\dots,M-1$ are independent. We set $A_m(t_i^{(m)})=x_i^{(m)}+jy_i^{(m)}$ where $t_i^{(m)}$ for $m=0,\dots,M-1$, $i=1,\dots,l_m$ is a set of arbitrary time values, and l_m are arbitrary integers. To show that the vector process $[A_0(t),\dots,A_{M-1}(t)]$ is Gaussian, we must show that $x_1^{(0)},\dots,x_{l_0}^{(0)},y_1^{(0)},\dots,y_{l_0}^{(0)},\dots,x_1^{(M-1)},\dots,x_{l_{M-1}}^{(M-1)}$, $y_1^{(M-1)},\dots,y_{l_{M-1}}^{(M-1)}$ are jointly Gaussian. Consider

$$\xi=\sum_{m=0}^{M-1}\sum_{i=1}^{l_m}(\omega_i^{(m)}x_i^{(m)}+\omega_{l_m+i}^{(m)}y_i^{(m)}) \tag{16}$$

where $\omega_i^{(m)}$, $\omega_{l_m+i}^{(m)}$ for $m=0,\dots,M-1$ and $i=1,\dots,l_m$ are arbitrary constants. Each of the inner sums in (16) is a Gaussian random variable, since each $A_m(t)$ is a Gaussian process. By the Cramer's theorem ξ is a Gaussian random variable, as a sum of independent Gaussian random variables. Now, by Definition 2.1, $x_1^{(0)},\dots,x_{l_0}^{(0)},y_1^{(0)},\dots,y_{l_0}^{(0)},\dots,x_1^{(M-1)},\dots,x_{l_{M-1}}^{(M-1)}$, $y_1^{(M-1)},\dots,y_{l_{M-1}}^{(M-1)}$ have joint Gaussian distribution since ξ is an arbitrary linear combination of $x_i^{(m)}$ and $y_i^{(m)}$ for $m=0,\dots,M-1$, $i=1,\dots,l_m$. Thus, the vector process $[A_0(t),\dots,A_{M-1}(t)]$ is complex Gaussian. Since any two processes $A_m(t)$ and $A_n(t)$ are uncorrelated, the cross-correlations are identically equal to zero, and thus, stationarity, ergodicity and properness of the vector process $[A_0(t),\dots,A_{M-1}(t)]$ are implied by the stationarity, ergodicity and properness of the component processes $A_m(t)$, $m=0,\dots,M-1$, respectively. ∎

III. Deriving FIR Channel Model

In this section we derive an approximate equivalent discrete time channel model for digital transmission. We start from the time varying channel model derived in Section II and also include the transmitter and receiver filters with impulse responses $g_t(t)$ and $g_r(t)$, respectively. We choose $g_t(t)$ and $g_r(t)$ such that $g_r(t)=g_t(-t)$ and $\eta=g_t*g_r$ to meet the Nyquist citerion. We model the data stream by

$$I(t)=\sum_n u_n\delta(t-nT) \tag{17}$$

where u_n is a sequence of complex signal points from the signal set. The signal at the output of the channel is

$$r(t)=\sum_n u_n\int c(\alpha,t)g_t(t-\alpha-nT)d\alpha \tag{18}$$

The output $y(t)$ of the receive filter due to the signal only is

$$y(t)=\sum_n u_n\int d\alpha\int d\gamma c(\alpha,\gamma)g_r(t-\gamma)g_t(\gamma-\alpha-nT) \tag{19}$$

Using $g_r(t)=g_t(-t)$ and $c(\tau,t)$ from (4), we get:

$$y(t)=\sum_n u_n\sum_m\int d\gamma A_m(\gamma)g_t(\gamma-t)g_t(\gamma-\tau_m-nT) \tag{20}$$

Assuming that $A_m(\gamma)$ does not change significantly in the interval over which $g_t(\gamma-t)$ is significant we obtain

$$y(t)=\sum_n u_n\sum_m A_m(t)\eta(t-\tau_m-nT) \tag{21}$$

By sampling this expression at times kT we obtain

$$y(kT)=\sum_n u_n\sum_m A_m(kT)\eta(kT-\tau_m-nT) \tag{22}$$

Setting:

$$q_\mu(kT)=\sum_m A_m(kT)\eta(\mu T-\tau_m) \tag{23}$$

where μ is an integer, we obtain the discrete time model:

$$y(kT)=\sum_n u_n q_{k-n}(kT) \tag{24}$$

The derived discrete time channel model is suboptimal. Even if A_m don't change with time the discrete time model obtained by symbol rate sampling is not equivalent to the continuous time model unless the receiver-transmitter filters are band-limited to $1/2T$. When channel varies in time, the sampling rate at channel output is to be increased by the amount of Doppler spread.

Theorem 3.1: The discrete channel model (24) is identical to the one obtained as the best mean square approximation of the concatenated transmitter filter - channel impulse response. Proof: Denote as $h(\tau, t)$ the time varying channel impulse response with transmitter filter included, i.e.

$$h(\tau, t) = \int c(\alpha, t) g_t(\tau - \alpha) d\alpha \tag{25}$$

Using the approach in [1] we compute an optimal mean square approximation $\hat{h}(\tau, t)$ to $h(\tau, t)$:

$$\hat{h}(\tau, t) = \sum_{\mu=-\infty}^{\infty} p_\mu(t) g_t(\tau - \mu T) \tag{26}$$

where

$$p_\mu(t) = \int_{-\infty}^{\infty} h(\tau, t) g_t(\tau - \mu T) d\tau = \int d\alpha c(\alpha, t) \eta(\alpha - \mu T) \tag{27}$$

For the assumed physical ray channel impulse response in (4)

$$p_\mu(t) = \sum_m A_m(t) \eta(\mu T - \tau_m) \tag{28}$$

Using the equivalent impulse response $h(\tau, t)$ we have

$$r(t) = \int h(\gamma, t) [\sum u_n \delta(t - nT - \gamma)] d\gamma = \sum_n u_n h(t - nT, t) \tag{29}$$

Setting $h(\tau, t) = \hat{h}(\tau, t)$ and using the expansion (26), we get

$$r(t) = \sum_n u_n \sum_\mu p_\mu(t) g_t(t - nT - \mu T) \tag{30}$$

and

$$\begin{aligned} y(t) &= \int r(\gamma) g_r(t - \gamma) d\gamma \\ &= \sum_n u_n \sum_\mu \int p_\mu(\gamma) g_t(\gamma - nT - \mu T) g_t(\gamma - t) d\gamma \end{aligned}$$

Assuming that $p_\mu(\gamma)$ does not change significantly during the interval over which $g_t(\gamma - t)$ is significant, we obtain

$$y(t) = \sum_n u_n \sum_\mu p_\mu(t) \eta(t - nT - \mu T) \tag{31}$$

Sampling this signal at the symbol rate and using the Nyquist property of η we subsequently get

$$\begin{aligned} y(kT) &= \sum_n u_n \sum_\mu p_\mu(kT) \eta(kT - nT - \mu T) \\ &= \sum_n u_n p_{k-n}(kT) \end{aligned} \tag{32}$$

Since $q_\mu(kT) = p_\mu(kT)$, (24) and (32) are identical. ∎

Omitting T's and setting $k - n \to n$ in (24) we get:

$$y_k = \sum_n q_n(k) u_{k-n} \tag{33}$$

Assuming that $q_n(k)$ are nonzero only for indices n between $-\nu_-$ and ν_+, introducing delay ν_-, and setting $\nu = \nu_- + \nu_+$ we obtain

$$y_k = \sum_{n=0}^{\nu} h_n(k) u_{k-n} + w_k \tag{34}$$

where $h_n(k) = q_{n-\nu_-}(k)$ and noise term is also added. Equation (34) describes a FIR channel model with time varying coefficients, which can be used in many applications. Note that k is the time index, and n is the lag (delay) index. Define the vector random process $\mathbf{h}(k) = [h_0(k), \cdots, h_\nu(k)]'$.

Theorem 3.2: For the assumed physical channel model from Section II and the approximate FIR discrete time model described by a vector random process $\mathbf{h}(k)$, obtained by sampling at the symbol rate and truncating the coefficients with negligible power, the vector process $\mathbf{h}(k)$ is a stationary and ergodic complex proper zero mean Gaussian process.

Proof: We use (23), which shows that $q_n(k)$ (and thus $h_n(k)$) for $k \in \mathbb{Z}$ are linear combinations of the processes $A_m(kT)$ for $k \in \mathbb{Z}$. To show that $\mathbf{h}(k)$ is a vector Gaussian process we use Theorem 2.3 and Definition 2.1. Properness of $\mathbf{h}(k)$ is a consequence of properness of $A_m(t)$.

To show that the vector process $\mathbf{h}(k)$, $k \in \mathbb{Z}$ is stationary we need to show that the correlations $\mathbb{E}[h_p(k+l) h_q^*(k)]$, $k \in \mathbb{Z}$, $0 \le p, q \le \nu$, depend on the lag l only. From (23) we get

$$\sum_{m=0}^{M-1} R_m^d(l) \eta(\tau_m - (p - \nu_-)T) \eta(\tau_m - (q - \nu_-)T) = R_{p,q}(l) \tag{35}$$

where we used the notation $R_m^d(l) = R_m(lT) = \mathbb{E}[A_m(kT + lT) A_m^*(kT)]$, and this is a function of l only due to the stationarity of $A_m(t)$. Therefore, the correlations are functions of the lag l only. Note that the crosscorrelations between $h_p(k+l)$ and $h_q(k)$ might be nonzero.

To show that $\mathbf{h}(k)$ for $k \in \mathbb{Z}$ is an ergodic vector process, it suffices to show that all the correlations $R_{p,q}(l)$, for $p, q = 0, \ldots, \nu$ approach zero as $|l| \to \infty$. According to (35) the FIR coefficient correlation functions are linear combinations of the correlations $R_m^d(l)$. From Theorem 2.2 $R_m^d(l) \to 0$ as $|l| \to \infty$ for all m, which in turn implies that $R_{p,q}(l) \to 0$ as $|l| \to \infty$ for all $p, q = 0, \ldots, \nu$. ∎

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