A New Coding Technique for the Transmission of UWB TH-PPM signals

Tudor .N. Durnea, PhD student

***Abstract* — Ultra Wide Band (UWB) Impulse Radio (IR) systems are implemented using a form of Pulse Position Modulation (PPM) for data encoding and Time Hopping (TH) for multiple access. A new code denoted as Totally Flipped Code (TFC) was designed for UWB TH-PPM signals and its p.s.d. calculated. The TFC code ensures a continuous p.s.d. with spectral nulls at bandwidth ends. The implementation of the coder and decoder requires minimum hardware additions on existing UWB TH-PPM systems and has no impact on the data rate.**

***Keywords* — Ultra Wide Band, Impulse Radio, Pulse Position Modulation, power spectral density, Totally Flipped Code.**

I. Introduction

THE Ultra Wide Band (UWB) systems were investigated from 1980 and regulated by FCC in USA from 2002 as short range data communication systems on a radio interface placed between 3.1 and 10.6 GHz [1]. The large bandwidth at high frequencies ensures high data rates and easy separation of direct component from multi-path versions. The bandwidth usage is free of charge. The requirements are for UWB systems to emit low level, almost flat power envelope signals. Any non-UWB system should be able to interpret the UWB signals' p.s.d. as Additive White Gaussian Noise (AWGN) for which their input filters would be perfectly capable to handle.

The initial candidate for UWB implementation was Impulse Radio (IR) – direct transmission of digital pulses to an antenna, without any analogue modulator. These systems were noted to have large discrete components which interfered with GPS, radars and mobile phone systems [2].

The following implementations were based on Spread Spectrum (SS) known techniques: Orthogonal Frequency - Division Multiplexing (OFDM) and Code Division Multiple Access (CDMA). The debates in IEEE 802.15 committee between the supporters of these two UWB versions resulted in a deadlock and the UWB as a standard was dropped because of lack of consensus in January 2006.

The document here-in presents a new code which could allow IR implementation to reach the initial goals of UWB systems: continuous, flat spectrum, low emitted energy high data rates and minimum hardware complexity.

The Totally Flipped Code was designed especially for UWB-IR systems but it can be used in any TH-PPM system.

Tudor N. Durnea is Ph.D. student at "Gh. Asachi" Technical University of Iasi, Romania, Faculty of Electronics and Telecommunications (Bd. Carol I, nr 11, RO-700506, phone:0040-727-390-913, e-mail: dtudor@zeta.etc.tuiasi.ro

II. Totally Flipped Code Presentation

The aggregate transmission of N_u UWB TH-PPM devices use a T_S symbol period to repeat N_S times a PPM pulse inside a T_f frame period on a chip period T_c selected randomly from N_h variants. The symbol $s_g(t)$ denotes the generator waveform, Δ the PPM modulation step, $c_{n\bullet N_S+m,k}$ the current TH position and $b_{n,k}$ the PPM position for user *"k"*.

$$u(t)=\sum_{n=-\infty}^{+\infty}\sum_{m=0}^{N_S-1}s_g(t-nT_S-mT_f-c_{nN_S+m}T_c-b_{n,k}\Delta) \quad (1)$$

The signal energy is accumulated at Pulse Repetition Rate (PRR) – inverse of symbol, frame, chip and PPM modulation step [3].

The Totally Flipped Code is inspired by the Partially Flipped Code [4]. The first step to eliminate the discrete spectral lines is to emit antipodal waveforms [5]. The choice is made that N_h is an even number and that TH pulses emitted on the first half of the frame period are positive, while the one on the last half are negative. The second step is to ensure that the Running Digital Sum (RDS) computed on TH pulses is bounded [6]. The Totally Flipped Code is used to achieve this requirement.

The aggregate UWB TH-PPM signal is described as:

$$u(t)=\sum_{n=-\infty}^{+\infty}\sum_{m=0}^{N_S-1}F_{nN_S+m,k}\bullet A_{nN_S+m,k}\bullet s_g(t-nT_S-mT_f-c_{nN_S+m}T_c-b_{n,k}\Delta) \quad (2)$$

where $A_{nN_S+m,k}$ is the sign based on TH position:

$$A_{nN_S+m,k}=\begin{cases}+1, c_{nN_S+m,k}<N_h/2\\ -1, c_{nN_S+m,k}>N_h/2\end{cases} \quad (3)$$

and $F_{nN_S+m,k}$ is the supplementary sign inversion inserted by TFC coder:

$$F_{n,k}=\begin{cases}-1 & , \quad 0<RDS_{n-1,k}\bullet A_{n,k}\\ F_{n-1,k} & , \quad 0=RDS_{n-1,k}\\ +1 & , \quad 0>RDS_{n-1,k}\bullet A_{n,k}\end{cases} \quad (4)$$

It is obvious that the sign of emitted pulses is not dependant on data content which is encoded in the PPM position. The TFC decoder must identify only the TH and PPM positions so it can discard the pulse sign.

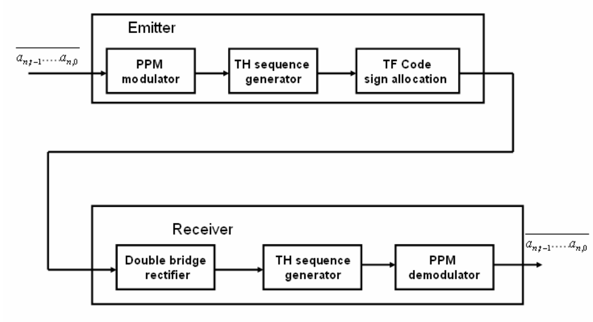

Fig. 1. TFC structure as implemented onto an UWB TH-PPM system.

The information is encoded in TH and PPM position while pulse sign is used only for p.s.d. aspect. The TFC receiver consists only of a double bridge rectifier to output the emitted TH-PPM signal.

III. CALCULATION OF UWB TH-PPM TFC P.S.D.

A. Basic Matrices

The Moore Finite State Machine (FSM) of TFC is presented below:

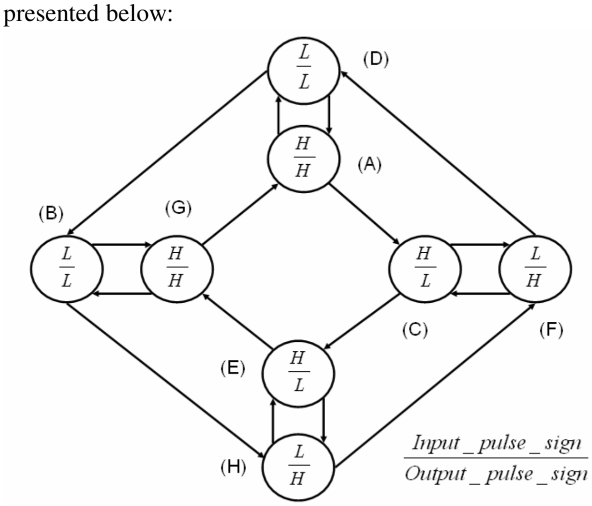

Fig. 2. FSM Moore for TFC coder.

The FSM Moore is built on macro-states. One such macro-state comprises all TH-PPM pulses which have the same sign (positive or negative). Supposing that PPM modulation is achieved on *"t"* bits, one macro-state has $2^t \bullet N_h/2$ states.

We denote by $P(\alpha = k \mid t)$ the probability that a random number α expressed on *"t"* bits has the value *"k"*. The transition probabilities between the states of two macro-states are described by the matrix:

$$Tran(i \bullet \frac{N_h}{2} + j, k \bullet \frac{N_h}{2} + l) = \frac{1}{N_h} \bullet P(\alpha = k \mid t) \quad (5)$$

$$i, k \in [0, 2^t - 1];\ j, l \in [0, \frac{N_h}{2} - 1]$$

The state stationary probabilities are derived after a simple calculus as:

$$P_A, P_B, P_C, P_D, P_E, P_F, P_G, P_H (i \bullet \frac{N_h}{2} + j) = = \frac{1}{4} \bullet \frac{1}{N_h} \bullet P(\alpha = j \mid t) \quad (6)$$

The stationary state probabilities of a macro-state are organized into the matrix:

$$Diag(i \bullet \frac{N_h}{2} + j, k \bullet \frac{N_h}{2} + l) = = \begin{cases} \frac{1}{4} \frac{1}{N_h} \bullet P(\alpha = k \mid t), & i = k \ \&\ j = l \\ 0, & i \neq k \parallel j \neq l \end{cases} \quad (7)$$

$$i, k \in [0, 2^t - 1];\ j, l \in [0, \frac{N_h}{2} - 1]$$

The correlation products of waveforms outputted from the states of two macro-states can be grouped in matrices of the form:

$$Z_{H\_H}(i \bullet \frac{N_h}{2} + j, k \bullet \frac{N_h}{2} + l) = s_g(t - n_1 T_S - m_1 T_f - j T_C - i\Delta) \bullet \bullet s_g(t - n_2 T_S - m_2 T_f - l T_c - k\Delta + \tau) \quad (8)$$

$$Z_{H\_L}(i \bullet \frac{N_h}{2} + j, k \bullet \frac{N_h}{2} + l) = s_g(t - n_1 T_S - m_1 T_f - j T_C - i\Delta) \bullet \bullet s_g(t - n_2 T_S - m_2 T_f - (l + \frac{N_h}{2}) T_c - k\Delta + \tau) \quad (9)$$

$$Z_{L\_H}(i \bullet \frac{N_h}{2} + j, k \bullet \frac{N_h}{2} + l) = = s_g(t - n_1 T_S - m_1 T_f - (j + \frac{N_h}{2}) T_C - i\Delta) \bullet \bullet s_g(t - n_2 T_S - m_2 T_f - l T_c - k\Delta + \tau) \quad (10)$$

$$Z_{L\_L}(i \bullet \frac{N_h}{2} + j, k \bullet \frac{N_h}{2} + l) = = s_g(t - n_1 T_S - m_1 T_f - (j + \frac{N_h}{2}) T_C - i\Delta) \bullet \bullet s_g(t - n_2 T_S - m_2 T_f - (l + \frac{N_h}{2}) T_c - k\Delta + \tau) \quad (11)$$

B. TFC characteristic matrices

All the TFC characteristic matrices have the dimensions $2^{t+2} \bullet N_h \times 2^{t+2} \bullet N_h$.

The matrix d of TFC stationary state probabilities is a diagonal matrix. All its diagonal elements are equal to $Diag$ matrix.

The matrix Π of TFC state transition probabilities has its non-null elements equal to $Tran$ matrix.

The matrix Z of state output waveform correlation has its elements of the form $Z_{H\_H}, Z_{H\_L}, Z_{L\_H}, Z_{L\_L}$ with the sign determined by the two macro-states' signs. Ex: $Z(\overline{A} \to \overline{D}) = -Z_{H\_L}$, $Z(\overline{H} \to \overline{G}) = Z_{L\_H}$.

It can be easily shown that:

$$\Pi^2 \bullet Z = 0 \quad (12)$$

All the TFC matrices and the relation presented above are valid for any statistical property of input data bits.

C. Mean Value and Autocorrelation Function

The UWB TH-PPM users are supposed to emit independent identically distributed (i.i.d.) data. The mean value of aggregated signal results as:

$$m_u(t) = N_u \sum_{n=-\infty}^{+\infty}\sum_{m=0}^{N_S-1}\sum_{i=0}^{2^{t+2}\bullet N_h-1} P(state_i)\bullet s_g(state_i) = 0 \qquad (13)$$

This is because any pulse can be emitted from 2 states with positive amplitude and from 2 other states with negative amplitude, all having the same probability.

The autocorrelation function is described using result (13) by the following formula:

$$R_u(t,t+\tau) = N_u \sum_{n_1,n_2=-\infty}^{+\infty}\sum_{m_1,m_2=0}^{N_s-1} Tr(d\Pi^{\left|(n_1-n_2)N_S+(m_1-m_2)\right|}Z) \qquad (14)$$

The result (12) holds that only the correlation of strictly successive TH pulses is non-zero. The autocorrelation function is split into three terms corresponding to TH pulses autocorrelation - $A(t,t+\tau)$, the correlation of pulses from the same UWB symbol - $B(t,t+\tau)$ - and the correlation of TH pulses from the border of two UWB symbols - $C(t,t+\tau)$:

$$R_u(t,t+\tau) = N_u \sum_{n=-\infty}^{+\infty}\sum_{m=0}^{N_s-1} Tr(dZ) + N_u \sum_{n=-\infty}^{+\infty}\sum_{\substack{m_1,m_2=0\\ |m_1-m_2|=1}}^{N_s-1} Tr(d\Pi Z) + N_u \sum_{\substack{n_1,n_2=-\infty\\ |(n_1-n_2)N_S+(m_1-m_2)|=1}}^{+\infty}\sum_{m_1,m_2=0}^{N_s-1} Tr(d\Pi Z) \qquad (15)$$

From the definitions of TFC characteristic matrices the Trace terms are expressed as:

$$Tr(dZ) = 4\bullet Tr(Diag\bullet Z_{H\_H} + Diag\bullet Z_{L\_L}) \qquad (16)$$

$$Tr(d\Pi Z) = -4\bullet Tr(Diag\bullet Tran\bullet Z_{H\_L}) - 4\bullet Tr(Diag\bullet Tran\bullet Z_{L\_H}) \qquad (17)$$

The aggregated signal is cyclo-stationary with the UWB symbol period T_S. We denote by $R_{sg}(\tau)$ the autocorrelation function of the generator waveform. The averaged mean on a symbol period for the constituent parts are.

$$\overline{A(\tau)} = \frac{N_u\bullet N_S}{T_S}\bullet R_{sg}(\tau) \qquad (18)$$

The following result is achieved using Appendix A:

$$\begin{aligned}\overline{B(\tau)} = &-\frac{N_u(N_S-1)}{T_S}\frac{1}{N_h^2}\sum_{v=-(\frac{N_h}{2}-1)}^{\frac{N_h}{2}-1}(\frac{N_h}{2}-|v|)\bullet\\ &\bullet\{R_{sg}(\tau-T_f-\frac{N_h}{2}T_c-vT_c)+R_{sg}(\tau-T_f+\frac{N_h}{2}T_c-vT_c)+\\ &+R_{sg}(\tau+T_f-\frac{N_h}{2}T_c-vT_c)+R_{sg}(\tau+T_f+\frac{N_h}{2}T_c-vT_c)\}\end{aligned} \qquad (19)$$

The first two members of the autocorrelation function are independent of the data content because the correlation products are achieved inside the same UWB symbol.

Appendix B introduces the statistical variable $Dif_{r,t}$ which represents the probability that two random numbers expressed on *"t"* bits are different by a constant offset *"r"*.

$$\begin{aligned}\overline{C(\tau)} = &-\frac{N_u}{T_S}\frac{1}{N_h^2}\sum_{v=-(\frac{N_h}{2}-1)}^{\frac{N_h}{2}-1}(\frac{N_h}{2}-|v|)\bullet\sum_{r=-(2^t-1)}^{2^t-1}Dif_{r,t}\bullet\\ &\bullet\{R_{sg}(\tau-T_S+(N_S-1)T_f-\frac{N_h}{2}T_c-vT_c-r\Delta)+\\ &+R_{sg}(\tau-T_S+(N_S-1)T_f+\frac{N_h}{2}T_c-vT_c-r\Delta)+\\ &+R_{sg}(\tau+T_S-(N_S-1)T_f-\frac{N_h}{2}T_c-vT_c-r\Delta)+\\ &+R_{sg}(\tau+T_S-(N_S-1)T_f+\frac{N_h}{2}T_c-vT_c-r\Delta)\}\end{aligned} \qquad (20)$$

D. Power spectral density of UWB TH-PPM coded TFC

The results (18), (19), (20) and Appendix C are used to calculate the p.s.d. of UWB TH-PPM signals coded TFC. We denote by $S_g(f)$ the power spectral density of generator waveform.

$$\begin{aligned}S_u(f) = &F[\overline{A(\tau)}+\overline{B(\tau)}+\overline{C(\tau)}] = \frac{N_uN_S}{T_S}S_g(f)-\\ &-\frac{N_u(N_S-1)}{T_S}S_g(f)\frac{4}{N_h^2}\frac{1-\cos(2\pi f\frac{N_h}{2}T_{C})}{1-\cos(2\pi fT_{C})}\bullet\\ &\bullet\cos(2\pi f\frac{N_h}{2}T_C)\cos(2\pi fT_f)-\\ &-\frac{N_u}{T_S}S_g(f)\frac{4}{N_h^2}\frac{1-\cos(2\pi f\frac{N_h}{2}T_{C})}{1-\cos(2\pi fT_{C})}\bullet\\ &\bullet\cos(2\pi f\frac{N_h}{2}T_C)\cos(2\pi f(T_S-(N_S-1)T_f)\bullet\\ &\bullet[Dif_{0,t}+2\bullet\sum_{r=1}^{2^t-1}Dif_{r,t}\bullet\cos(2\pi f\bullet r\Delta)]\end{aligned} \qquad (21)$$

It can be easily shown that

$$S_u(0) = \frac{N_uN_S}{T_S}S_g(0)-\frac{N_u(N_S-1)}{T_S}S_g(0)-\frac{N_u}{T_S}S_g(0) = 0 \qquad (22)$$

The components of the spectral envelope of UWB TH-PPM TFC coded signals are presented below.

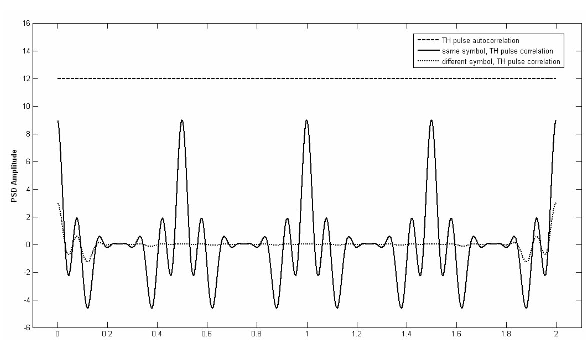

Fig. 3. p.s.d. components of TFC UWB TH-PPM with $t=2, N_h = N_S = 4, N_u = 3$

The contribution of TH pulses from different symbols is reduced because of order of magnitude ($N_S - 1$ smaller than the other components) and because of the distribution of PPM positions. The complete p.s.d. is presented below:

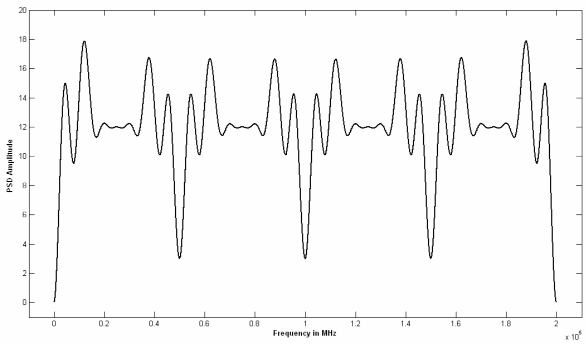

Fig. 4. p.s.d. of TFC UWB TH-PPM

The p.s.d. comprises short flat areas between formations of 4 small maxima and one deep minima.

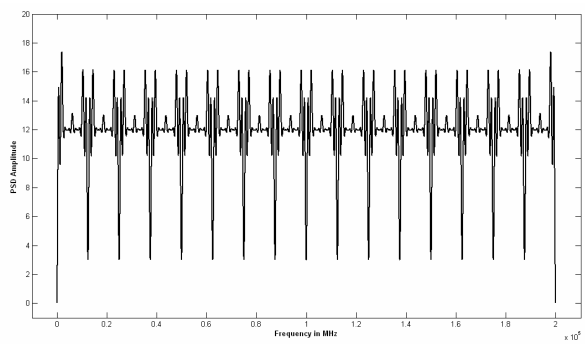

Fig. 5. p.s.d. of TFC UWB TH-PPM with $t=4, N_h=6, N_S=4, N_u=3$

The influence of TH-PPM parameters on resultant p.s.d. is as follows: N_u, N_S linearly increase the amplitude of p.s.d. and keep its overall aspect. Increasing N_h induces a small reduction in the size of p.s.d.'s local maxima and minima. Increasing *"t"* - the number of PPM bits – increases the number of maximum and minimum formations up to 2^t and makes them sharper.

The mean value of TFC coded UWB TH_PPM p.s.d. is equal to the value of the p.s.d. for random sign TH-PPM pulses – which is regarded as the optimum case [7], [8].

IV. CONCLUSION

A new code for the transmission of UWB TH-PPM signals was designed and investigated. The hardware additions required by the TFC coder and decoder are minimal. There is no reduction in user or system data rate. The generated spectrum is continuous with spectral nulls at bandwidth ends. The spectrum aspect is independent of data bit statistical properties. The spectrum magnitude is independent of the number of bits used in PPM modulation. The spectrum presents small local maxima and deep local minima. The TFC code can easily be implemented in any UWB TH-PPM system to eliminate discrete spectral lines and introduce spectral nulls at bandwidth ends.

APPENDIX

A. It can be easily shown that the sums corresponding to Time Hopping positions can be reduced to:

$$\sum_{j_1=0}^{\frac{N_h}{2}-1}\sum_{j_2=0}^{\frac{N_h}{2}-1} R_{sg}(\tau-(j_1-j_2)T_C) = \sum_{v=-(\frac{N_h}{2}-1)}^{\frac{N_h}{2}-1}\left(\frac{N_h}{2}-|v|\right)R_{sg}(\tau-vT_C)$$

B. The mathematical definition of the *Dif* variable is:

$$Dif_{r,t} = \sum_{k=0}^{2^t-1-r} P(A_i=k+r \mid t)\bullet P(A_j=k \mid t)$$

Some important properties of the *Dif* variable [9] are presented below.

P1 $\sum_{r=-(2^t-1)}^{2^t-1} Dif_{r,t}=1$ P2 $Dif_{r,t}=Dif_{-r,t}, \forall r,t$

P3 $$\begin{cases} Dif_{0,t}=[1-2p(1-p)]^t \\ Dif_{2g,t}=[1-2p(1-p)]\bullet Dif_{g,t-1} \end{cases}, \quad g\in[0.2^{t-1}-1]$$

P4 $$\begin{cases} Dif_{2^t-1,t}=p^t(1-p)^t \\ Dif_{2g+1,t}=p(1-p)[Dif_{g,t-1}+Dif_{g+1,t-1}] \end{cases}, \quad g\in[0.2^{t-1}-2]$$

C. The initial sum from Appendix A is translated into frequency domain as:

$$\frac{N_h}{2}+2\sum_{v=1}^{\frac{N_h}{2}-1}\left(\frac{N_h}{2}-v\right)\cos(2\pi fvT_C)=2\sum_{v=0}^{\frac{N_h}{2}-1}\left(\frac{N_h}{2}-v\right)\cos(2\pi fvT_C)-\frac{N_h}{2}$$

$$=2\sum_{k=0}^{\frac{N_h}{2}-1}\sum_{v=0}^{k}\cos(2\pi fvT_C)-\frac{N_h}{2}=\frac{1-\cos(2\pi f\frac{N_h}{2}T_C)}{1-\cos(2\pi fT_C)}$$

This function is continuous and its limit cases have a maximum value of:

$$\lim_{f\to 0}\frac{1-\cos(2\pi f\frac{N_h}{2}T_C)}{1-\cos(2\pi fT_C)}=\left(\frac{N_h}{2}\right)^2$$

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