The Impact of Subcarrier Spacing on Performance of WiMAX DL-FUSC in the Nonergodic Case

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Abstract— **Recent research on performance of traffic channels for mobile services in OFDMA systems has shown that multiuser diversity cannot be efficiently exploited due to highly dynamic propagation channels and high cost of signaling overhead. In case of mobile services, system outage capacity is maximized when traffic channels are configured with maximum frequency diversity. One such traffic channel with distributed subcarrier permutation, intended for mobile services is the DL-FUSC channel used in WiMAX IEEE 802.16 standards. The permutation formula that defines subcarrier positions for DL-FUSC allows non-equally spaced subcarriers. This could be a cause of potential performance degradation of DL-FUSC compared to the case where subcarriers are equally spaced. We present comparison of performance of DL-FUSC traffic channel to performance of properly designed theoretical traffic channel models in order to obtain insight in the impact of non-equal subcarrier spacing on performance.**

Keywords – WiMAX, DL-FUSC, OFDMA, outage capacity, outage probability.

I. INTRODUCTION

In recent years, many broadband wireless network standards are accepting OFDMA (Orthogonal Frequency Division Multiple Access) as a multiple access scheme of choice compared to single carrier solutions on frequency selective channels, due to the avoidance of the use of equalizers. These standards include WiMAX (Worldwide Interoperability for Microwave Access) standards presented in [1] and [2]. OFDMA networks support multiuser communication where each OFDMA symbol consists of many subcarriers in frequency domain and these subcarriers are assigned to users in many different ways. Many researchers have investigated the resource allocation problem in OFDMA systems. The research mainly branched in two areas: in the area of information theory providing results for channel capacity for multiuser systems over time-varying fading channels, when full channel state information (CSI) or partial channel state information is known ([3]), and, finding efficient algorithms for resource allocation ([4]-[7]).

In contrast to considering each carrier as a resource allocation unit, a more practical approach has been taken in [8] and [9], where a number of subcarriers are grouped in a single unit called traffic channel. In multiuser systems each user experiences different fading conditions, this leading to possibility of exploiting multiuser diversity and increasing the overall system capacity. This requires knowledge of CSI at the transmitter that must be periodically updated. Obviously, multiuser diversity can be fully utilized on slowly varying frequency selective fading channels. In this case, utilizing multiuser diversity implies using traffic channels made of adjacent subcarriers. Thus, the achievable data rate variation among users is the highest. This is the case when fixed/portable services are provided to the user. In case of mobile services, fast varying frequency selective fading channels are expected, where periodic update of CSI requires considerable increase in signaling overhead. Here, traffic channels are designed in such a way that subcarriers are distributed throughout the OFDM symbol to provide both frequency diversity and minimum variation of the achievable data rate among users.

In the case of WiMAX DL-FUSC (Downlink Fully Used SubChannelization) traffic channel specified in [1] and [2], all the data subcarriers are used to create traffic channels. DL-FUSC is a distributed multicarrier traffic channel that uses a permutation scheme designed to allow non-equally spaced subcarriers, in order to avoid inter-cell interference. This might cause a possible performance degradation compared to distributed traffic channels with equally spaced subcarriers. In this paper we compare the performance of DL-FUSC with the performance of properly designed multicarrier traffic channel permutation schemes that allow for non-equally spaced subcarriers. We use the theoretical approach in [8], i.e. we compare different schemes by evaluating outage capacity. Our analysis shows that outage performances of all these traffic channels virtually do not differ.

Paper is organized as follows. In Section II wireless channel model is described and in Section III different traffic channel models are developed. Outage capacity expression is given in Section IV, followed by simulation results and conclusion in Sections V and VI, respectively.

II. WIRELESS CHANNEL MODEL

We assume that signal bandwidth is larger than channel coherence bandwidth, so that we have frequency selective channel. We also assume under-spread model of time-varying frequency selective channel with $T_m \ll T_c$, where T_m is the delay spread of the channel and T_c is channel coherence time. Although we model slowly varying frequency selective

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channel we assume that mobile services have time dynamics on the wireless channel that is higher than for fixed/portable services and where to maintain full CSI signaling is improbable. Thus we have CSI available at the receiver only. We assume that the OFDM symbol duration is less than channel coherence time and that cyclic prefix is added of size larger than channel memory.

We model the broadband channel in a Rayleigh fading environment as γ uncorrelated paths with delays normalized by the symbol duration T_s : $\tau_0, \tau_1, \dots, \tau_{\gamma-1}$ $(\tau_0 = 0)$. Furthermore we assume that the gains of the γ uncorrelated paths are independent complex Gaussian random variables: $\boldsymbol{\alpha} = [\alpha_0, \alpha_1, \dots, \alpha_{\gamma-1}]^T$, having independent real and imaginary parts with zero mean and variance $\frac{\sigma_i^2}{2}$, $i = 0,1, \dots, \gamma-1$. We assume OFDM block size of N and channel frequency response vector $\boldsymbol{h} = [h_0, h_1, \dots, h_{N-1}]^T$, where $h_n = \sum_{i=0}^{\gamma-1} \alpha_i \exp\left[-\frac{j(2\pi\tau_i n)}{N}\right]$, $n = 0,1, \dots, N-1$. Denoting $w_N = \exp\left[-j\left(\frac{2\pi}{N}\right)\right]$, we have $\boldsymbol{h} = \boldsymbol{W}\boldsymbol{\alpha}$ where $\boldsymbol{W}_{N\times\gamma} = [(w_N)^{n\tau_i}\colon\ n = 0,1, \dots, N-1;\ \ i = 0,1, \dots, \gamma-1]$. It can be shown that $h_n, n = 0,1, \dots, N-1$, are identically distributed, correlated zero mean complex Gaussian variables with equal variances $\sum_{i=0}^{\gamma-1}\left(\frac{\sigma_i^2}{2}\right)$ on real and imaginary parts. If x_n and y_n are the transmitted and received symbols on the n^{th} subcarrier, under the assumption that cyclic prefix is larger than the length of the channel impulse response, we have $y_n = h_n x_n + v_n, n = 0,1, \dots, N-1$. Here v_n is a white Gaussian noise with zero mean and variance δ^2 . Introducing vector notation $\boldsymbol{x} = [x_0, x_1, \dots, x_{N-1}]^T$, $\boldsymbol{h} = [h_0, h_1, \dots, h_{N-1}]^T$, $\boldsymbol{v} = [v_0, v_1, \dots, v_{N-1}]^T$ and $\boldsymbol{y} = [y_0, y_1, \dots, y_{N-1}]^T$ we have $\boldsymbol{y} = \boldsymbol{h}\odot\boldsymbol{x} + \boldsymbol{v}$. Here $\odot$ represents element by element multiplication. If we assume that $\Omega_l = \{l_1, l_2, \dots, l_M\}$ is the set of subcarrier indices of a particular traffic channel l, then $\boldsymbol{h}_l = [h_{l_1}, h_{l_2}, \dots, h_{l_M}]^T$ is the channel vector corresponding to the l^{th} traffic channel. In order to focus on the effect of the different traffic channel configurations, we normalize the downlink channel power to unity $E\{|h_n|^2\} = 1$ and remove the propagation loss factor.

III. TRAFFIC CHANNEL MODELS

Our goal is to compare DL-FUSC performance to the performance of theoretical traffic channel models. The traffic channels compared are obtained as follows:

A. *DL-FUSC (OFDMA 2048)*

DL-FUSC is part of the [1] and [2] standard and is chosen here since it is the most basic distributed traffic channel permutation spanning over a single OFDMA symbol. The OFDMA symbol structure is constructed using pilots, data and zero subcarriers. After allocating the appropriate pilot and zero subcarriers, all the remaining subcarriers are used as data subcarriers that are further divided into traffic channels (in the standard traffic channels are known as sub channels). There are 24 fixed pilots and 142 variable pilots, the variable pilots changing from one FUSC symbol to another.

The data subcarriers are divided into traffic channels, each consisting of 48 subcarriers distributed throughout the OFDMA symbol. Details are provided in [1] pp.564-567.

The subcarriers are chosen using permutation formula and are allocated out of the data subcarrier domain. The data subcarrier domain includes 1536 subcarriers, which are the remaining sub carriers after removing all pilot and zero subcarriers including the DC subcarrier from the subcarrier domain (0-2047).

To allocate data subcarriers to traffic channels, they are partitioned into groups of contiguous subcarriers. Each traffic channel consists of a single subcarrier from each of these groups. The number of groups is therefore equal to the number of subcarriers per traffic channel and is denoted as $N_{subcarriers}$. The number of subcarriers in a group is equal to the number of traffic channels and is denoted as $N_{subchannels}$. The number of data subcarriers is thus equal to $N_{subcarriers} \times N_{subchannels}$. The exact partitioning into traffic channels is according to the following permutation formula

$$\begin{aligned} subcarrier(k,s) = {} & N_{subchannels} \cdot n_k + \\ & +\{p_s[n_k \bmod N_{subchannels}] + \\ & +DL\_PermBase\} \bmod N_{subchannels} \end{aligned} \qquad (1)$$

where

$subcarrier(k,s)$	is the index of subcarrier k in subchannel s,
s	is the index of a subchannel, from the set $[0, \dots, N_{subchannels}-1]$,
n_k	$= (k + 13\cdot s) \bmod N_{subcarriers}$, where k is the subcarrier-in-subchannel index from the set $[0, \dots, N_{subcarriers}-1]$,
$N_{subchannels}$	is the number of subchannels,
$p_s[j]$	is the j-th term in the series obtained by rotating the basic permutation sequence cyclically to the left s times,
$DL\_PermBase$	is an integer ranging from 0 to 31, which identifies the particular BS segment and is specified by MAC layer (is set to preamble *IDcell* in the first zone and determined by the DL_MAP for other zones),
$X_{mod(k)}$	is the remainder of the quotient X/k.

The remaining parameters are presented in the table 311 on page 565 of [1].

As can be seen, a DL-FUSC traffic channel is obtained choosing a single subcarrier from each group of subcarriers. In forming the DL-FUSC channel, the permutation formula also takes into account minimizing number of hits among the same sub channel number in different cells (possibly neighboring Base Stations). This is explained in [10] as a technique to reduce interference in neighboring cells. We use 20 MHz bandwidth, so that there are 2048 subcarriers. Thus, DL-FUSC has $N_{subchannels}$=32 subchannels with $N_{subcarriers}$=48 subcarriers each, for a total of 1536 data subcarriers.

B. Theoretical Traffic Channel Models

Here we propose three properly designed theoretical traffic channel models that we use to compare the performance of DL-FUSC with. These theoretical traffic channel models do not use complicated permutation formulas that take into account other effects, such as reduction of interference from neighboring cells, but present clear cases of traffic channel configurations with different spacing among subcarriers.

First model is the same as the one used in [8] of type (M,1). Notation (M,1) means that there are M subcarriers per traffic channel, distributed over the data subcarriers in the OFDMA symbol. Data subcarriers are divided into groups with $L=N/M$ subcarriers in each group. L is also the number of traffic channels. So, M and L correspond to $N_{subcarriers}$ and $N_{subchannels}$, previously defined for DL-FUSC. We choose subcarriers regularly from within every group. Traffic channel number one takes the first subcarrier of every group, traffic channel number two takes the second subcarrier of every group, and so on. In this way we obtain regular spacing among subcarriers, i.e. the subcarriers are equally spaced N/M subcarriers apart. Since we use a total of 48 subcarriers per traffic channel, we refer to this model as (48,1).

Second model is formed with some freedom in obtaining subcarrier indices and, thus, results in non-equal spacing between subcarriers. First, data subcarriers in the OFDMA symbol are split into M equal groups each consisting of L subcarriers. In each group we randomly choose a single subcarrier out of $L=32$ possible subcarriers. In this way we obtain traffic channel with $M=48$ subcarriers and non-equal subcarrier spacing. From the way this traffic channel is formed we can see that it is very close to the way that DL-FUSC is formed. This model is named RG (**R**andom Subcarrier per **G**roup).

Third model is formed with complete freedom in the choice of subcarrier indices from all 1536 data subcarriers in the OFDMA symbol, i.e. we randomly choose $M=48$ out of 1536 data subcarriers. This model is named R (**R**andom Subcarrier per Symbol).

Figure 1 shows a sample look of all discussed traffic channel models.

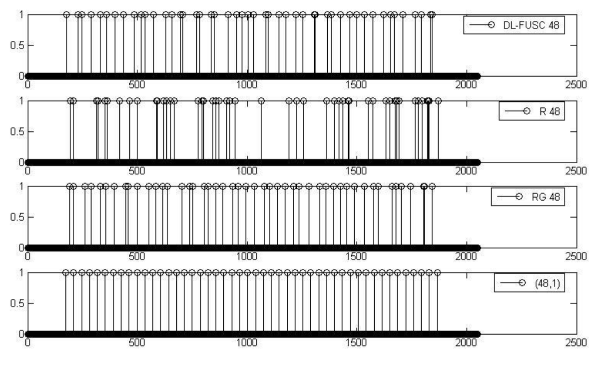

Figure 1. Sample look of DL-FUSC, R-48, RG-48 and (48,1)

As we can see from Figure 1 DL-FUSC is similar to the RG model. Since DL-FUSC and RG have some freedom in choosing subcarriers, there is variation in subcarrier spacing compared to the regular (48,1) channel model. R model has complete freedom in choosing subcarriers which results in higher standard deviation of the subcarrier spacing.

IV. Traffic Channel Performance Analysis

Performance analysis is done invoking the concepts of outage capacity and outage probability. Distributed subcarrier models as described above are suitable for mobile applications when high fading dynamics of the wireless channel is assumed and CSI is available at the receiver only. In this case it is possible to study channel capacity for two types of services: delay-nonsensitive and delay-constrained services. Providing that the channel vector is a random process, we can define ergodic capacity and outage capacity respectively ([3]). Investigations done in [8] show that the traffic channel configuration has no effect on the ergodic capacity but it does have influence on the outage capacity in the nonergodic case. This last is the case of interest in most practical scenarios. In the nonergodic case the coding spans a finite number of OFDM blocks and Shannon capacity does not exist since the mutual information is a random variable depending on $\boldsymbol{h}$. In this case the concept of outage capacity is invoked. It is also referred to as ε-capacity. This is the capacity guaranteed for $(100-\varepsilon)\%$ of the channel realizations. The outage probability for a given rate r, $P_{out}(r)$, is defined as the probability that the mutual information I falls bellow r: $P_{out}(r) = P(I < r)$. Accordingly, the outage capacity $r(\varepsilon)$ is the largest r such that the outage probability is less than a given probability ε and we can write $r(\varepsilon) = sup_{\{r:P_{out}(r)<\varepsilon\}}\, r$. For simplicity we assume that independent coding/decoding is performed on each traffic channel. In this case the traffic channel configuration only affects the amount of correlation between the subcarriers within the traffic channel. From [8] the mutual information on the traffic channel l can be computed from

$$I_l = \frac{1}{M}\sum_{m=1}^{M} log_2\left(1+\frac{\left\|h_{l_m}\right\|^2}{N_0}\right) = \frac{1}{M} log_2 \prod_{m=1}^{M}\left(1+\frac{\left\|h_{l_m}\right\|^2}{N_0}\right). \quad (2)$$

V. Simulation Results

In this section we provide simulation results for the traffic channels introduced in III. For simulation we use the wireless channel model described in II and the power delay profile for TU (Typical Urban) area from [11]. For the OFDMA system we assume frequency channel bandwidth of $B = 20MHz$ and $N = 2048$ ([2]). To get statistical relevance, we use 1000 realizations of COST 259 TU.

Simulation results are shown in Figure 2 and Figure 3.

From Figure 2 and Figure 3 we can see that there are almost no differences in performance for different subchannel configurations. We conclude that DL-FUSC configuration, which uses permutations in order to reduce the effect of inter-cell interference, and resembles our model RG, does not degrade performance compared to the case of equally spaced subcarriers, usually analyzed in literature. Figure 2 shows that small deviations exist in the region of low and high values of outage probability only for model R (subcarriers in a traffic channel distributed randomly throughout the entire available bandwidth). The same can be seen from Figure 3 for low values of outage probability. But, even in this case, performance is not reduced significantly.

VI. CONCLUSION

We present comparison of performance of DL-FUSC traffic channel with performance of three properly designed theoretical models of traffic channels. We show that, when nonergodic case is considered, performance differences in terms of outage capacity are almost negligible for all four traffic channel models.

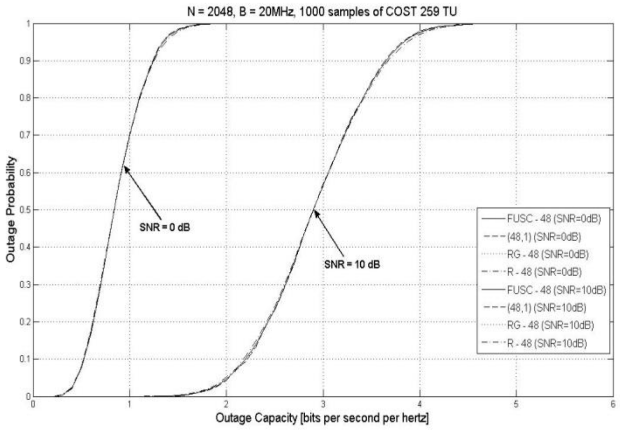

Figure 2. Outage Probability versus Outage Capacity for SNR=0 dB and SNR=10 dB

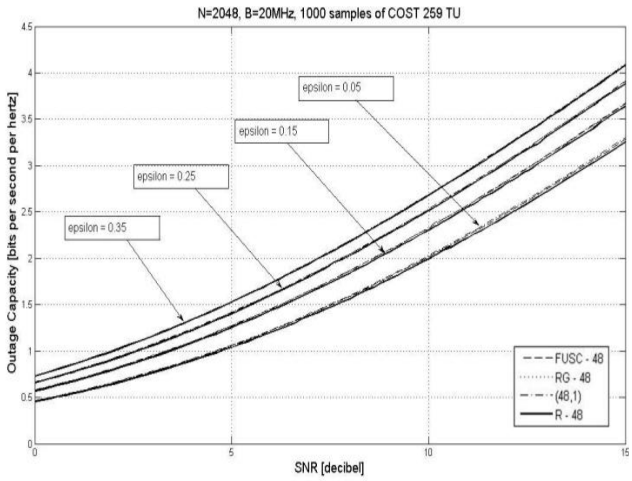

Figure 3. Outage Capacity versus SNR

VII. REFERENCES

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