Performance Evaluation of STBC MIMO Systems with Linear Precoding

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***Abstract* — It is known that transmit channel side information (CSIT) is used to enhance the performance of space-time block code based multi-antenna communication links. In this paper we analyze how transmission algorithms can be adapted to the channel condition based on the degree of the available CSIT and the system diversity order. The precoding design criteria considered is minimizing the average pairwise error probability.**

***Keywords* — CSIT, precoding, STBC, waterfilling.**

I. Introduction

Conventional SISO (Single Input Single Output) systems are limited by the multipath propagation and interference, so they cannot satisfy the demand for high data rates and better quality systems [1]. The benefits of MIMO (Multiple Input Multiple Output) techniques are well established: linear growth in transmission rate with the minimum number of antennas, enhance in the link reliability and coverage, efficient use of bandwidth, are all obtained without additional radio resource requirements, like bandwidth or more transmit power. The only demand is that the receiver has perfect knowledge of the channel state information (CSIR – channel state information at the receiver).

Exploiting channel state information at the transmitter (CSIT) can provide further enhancement in the performance of a MIMO system, regarding both the channel capacity and the system error performances, even if the channel is spatially correlated. These adaptive technique allows the transmitter to adapt to the propagation condition [2].

The rest of the paper is organized as follows: in Section II the system model with space-time block coding and CSIT is presented. In Section III we introduce the design of the linear precoder for three different types of CSIT (full, mean, covariance), under the constraint of a fix value transmit-power. Section IV contains the results of the simulations and in Section V the conclusions are drawn.

II. System Model

A frequency-flat Rayleigh fading MIMO wireless channel with N_T transmit and N_R receive antennas is considered. The system is encoded with a space-time block code: the incoming bits b_i are mapped onto a vector $s = [s_0, s_1, ..., s_{N_S-1}]^T$, where s_i is a symbol from a uniform signal constellation such as M-QAM, M-PSK or M-PAM [3]. The generated symbols are encoded in two dimensions, space and time, according to a OSTBC design matrix $C(s)[N_S xN]$, where N_S and N are the time and space dimensions. The codeword matrix [4,p85] is then processed by a precoder $F[N_T xN_S]$, designed according to the available CSIT. The $N_R xN$ received signal becomes:

$$Y = HFC(s) + n \qquad (1)$$

where $n[N_R xN]$ is the additive white noise and $H[N_R xN_T]$ is the channel matrix.

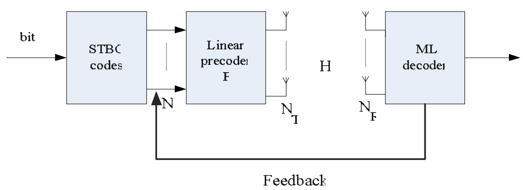

Fig. 1. STBC MIMO system with CSIT

At the receiver, which is assumed to have perfect knowledge of channel state, a MLD (Maximum Likelihood Decoding) detection is performed in order to estimate the transmitted symbols [5]:

$$\overline{C} = \arg\min_{C} \| Y - HFC \|_F^2 \qquad (2)$$

The design of the precoding matrix depends on the degree of available CSIT and the performance criteria that is considered: maximizing the system ergodic capacity, minimizing the pair-wise error probability (PEP), the symbol error probability (SER) or the mean squared error (MSE) [5]. For the simulations in this article, minimizing the pair-wise error probability is considered.

III. Precoding Design

The general form of a linear precoder is given bellow:

$$F = U_F D V_F^H \qquad (3)$$

which is the singular value decomposition of the matrix.

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The left singular vectors U_F give the orthogonal beam directions, the beam power loadings are the squared singular values D^2 and, V_F, the right singular vectors, is the input shaping matrix as in [5]. The constraint that the matrix has to satisfy is that the sum of power over all beams must be constant, $tr(FF^*) = 1$.

A. Full CSIT

If the entire channel matrix is available at the transmission the precoder is based on the channel matrix H and on the input codeword covariance matrix Q as in [5]:

$$F = V_H \Lambda_f U_Q \tag{4}$$

where $V_H[N_R x N_R]$ is obtained by singular value decomposition of the channel matrix and $U_Q[N_S x N_S]$ is obtained by eigenvalue decomposition of the covariance matrix Q. The optimal power allocation, given in [5], is through water-filling and the value depends on the eigenvalues of the input codeword covariance and of the channel:

$$p_i = (\lambda - \frac{N_0}{\lambda_i(HH^H)\lambda_i(Q)})_+ \tag{5}$$

where λ is the Lagrange multiplier chosen to satisfy the power constraint and N_0 is the noise power per spatial dimension.

B. Mean CSIT

For this type of feedback the MIMO channel matrix is given by:

$$H = \overline{H} + \Xi \tag{6}$$

where $\overline{H}\ [N_R x N_T]$ is the CSIT estimated at the transmission and $\Sigma[N_R x N_T]$ is the CSIT error matrix. These two matrices are uncorrelated $E[\overline{H}\Xi^H] = 0$, their entries are zero-mean complex Gaussian with variances $\sigma_e^2 + \sigma_h^2 = 1$. The optimal precoder is given in [6]:

$$F = U_h \Lambda_f V_e^H \tag{7}$$

where $U_h[N_T x N_T]$ is obtained by singular value decomposition of the matrix $\overline{H}$ and $V_e[N_s x N_s]$ is obtained by eigenvalue decomposition of the codeword error matrix $E(m,n) = [s_m - s_n]$, which is the error probability of choosing the nearest space-time codeword s_n instead of the transmitted codeword s_m [6]. The power allocated to each sub-channel is computed based on the eigenvalues of the estimated channel matrix $\Lambda_h[N_R x N_T]$ and of the codeword error matrix $\Lambda_e[N_s x N_s]$, and also on the noise variance σ_n^2. The value is given by:

$$p_k^* = (\frac{\sqrt{\frac{\lambda_{h,k}^2 \lambda_{e,k}}{\lambda}} - 1}{\sigma_n^2 \lambda_{e,k}})_+ \tag{8}$$

where λ is the Lagrange multiplier.

C. Covariance CSIT

When the channel covariance matrix is available at the transmitter, the optimal precoder desing that minimizes the maximum pairwise error probability is given in [7] as:

$$F = \frac{1}{\sqrt{\eta}} U_T \sqrt{B} \Phi \tag{9}$$

The precoder matrix depends on the correlation between the transmit antennas $R_T = U_T \Lambda_T U_T^H$, on the noise variance σ_n^2 and a scaling factor μ_{kl} that depends on the codeword matrix at the transmission C_k and the codeword matrix after detection C_l:

$$\eta = \frac{\mu_{\min}}{4\sigma_n^2} \tag{10}$$

$$\mu_{\min} = \arg\min_{\mu_{kl}} \{\mu_{kl} I = (C_k - C_l)(C_k - C_l)^H\} \tag{11}$$

The power allocation is an extension of the waterfilling problem to two dimensions (if the correlation between the receive antennas is also considered), and the matrix is solution is given in [7]:

$$B_{opt} = \arg \max_{\substack{B \geq 0 \\ tr(B) = \eta N_T}} \det[I_{N_R} \otimes B + \Lambda_R^{-1} \otimes \Lambda_T^{-1}] \tag{12}$$

which is equivalent with finding non-negative b_m values that maximize $\prod_{m=1}^{N_T} \prod_{n=1}^{N_R} (b_m + \lambda_{rn}^{-1} \lambda_{tm}^{-1})$, Λ_T and Λ_R are the eigenvalues of the transmit, respectively receive, antennas correlation matrices. For a MISO system the complexity of the computation is significantly reduced, $b_i = \max(\nu - \lambda_{ti}^{-1}, 0)$, $i = 1,..,N_T$ and ν is a constant chosen to satisfy the power constraint.

IV. SIMULATION RESULTS

We provide simulations for MIMO systems with a varying number of transmit/receive antennas, based on different types of CSIT. The transmit symbols are uniformly distributed based on a M-PSK/M-QAM constellations. The transmit power, across all transmit antennas, is set to one.

The purpose of these simulations is to see when it is efficient to use CSIT and which is the amount of feedback information necessary to obtain certain performance, for different configurations of MIMO systems.

The simulations are realized using Matlab 7. The analyzed parameter is the bit error rate.

For the first simulations that were done we considered no receive diversity, a system with two transmit antennas and one receive antenna, encoded with the rate one Alamouti space-time code. By considering this configuration it can be analyze how the performance of the MIMO system can be improved by acting only at the transmission side. For the first two types of CSIT feedback the transmit antennas are supposed to be uncorrelated.

In Fig. 1. there are the results obtained if the transmitter is assumed to have full channel state knowledge. With a small diversity order, equal to 2, there is a precoding gain of 5dB for the entire SNR (Signal to Noise Ratio) domain that is analyzed.

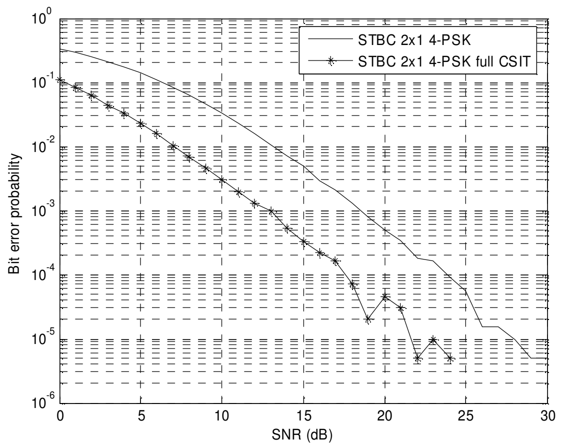

Fig. 2. Precoding gain for a 2x1 STBC MIMO system with full CSIT

If the transmitter has only statistical information about the channel, mean based estimation with small CSIT error $\sigma_e^2 = 0.01$, the precoding gain is smaller, about 3dB, but it still outperforms the transmission with no CSIT as it can be observed in Fig. 3.

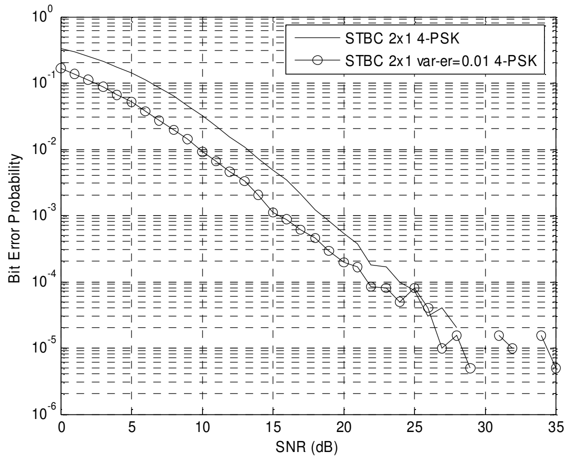

Fig. 3. Precoding gain for a 2x1 STBC MIMO system with mean CSIT

In the next simulation we analyze the performance of a covariance based CSIT transmission with one receive antenna employing a rate one space-time block code. First of all it can be noticed that the spatial correlation dramatically affects the performance of a STBC transmission with no CSIT. For low values of SNR, smaller that 6dB, the performances of non-coded transmission are similar, but above this threshold, diversity is essential and the non-correlated systems outperforms the transmission when there is a correlation of $\rho = 0.9775$ between the two transmit antennas. It can be noticed that the BER curve is different, as the diversity order is different.

If the transmit antennas are correlated ($\rho = 0.9775$), but the system benefits from CSIT, there is a precoding gain of about 7dBs for the entire SNR region. So if the channel is correlated, adapting the transmission based on available covariance CSIT leads to a significant error performance improvement.

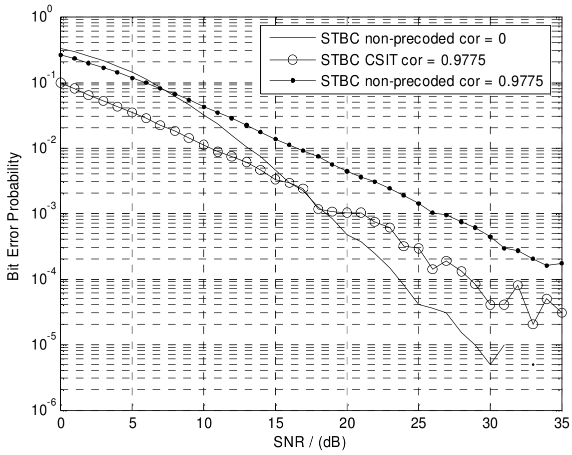

Fig. 4. Precoding gain for a 2x1 STBC MIMO system with covariance CSIT

In TABLE 1 there are the values of the signal to noise ratio needed for various configurations of MIMO system with and without CSIT to ensure a specified performance regarding the bit error rate. It can be observed here how the MIMO transmission with CSIT is influenced by the receive and transmit diversity.

By increasing the number of receive antennas, $N_R = 2$, the SNR gain is by about 10dBs for both transmissions with and without CSIT. In what fallows we will compare the *2x2* MIMO system with the systems obtained by adding additional receive and transmit antennas: *2x4* and *4x2*.

For the system with no CSIT, receive diversity is more important to achieve better error performance. A rate one transmission is 3dB better than the transmission with the same diversity order obtained with four transmit antennas. This observation is not true for the systems with CSIT, especially if the precoder design is based on statistical CSIT. The *2x4* mean based configuration requires a SNR=9dB for $BER = 10^{-5}$ while the *4x2* system only requires 7dB to obtain the same performance, but the limitation of the orthogonal space-time block code with 4 transmit antennas is its rate of ½. The error performance improvement is due to higher precoding gain

TABLE 1: PERFORMANCES RESULTS FOR DIFFERENT DIVERSITY ORDER MIMO SYSTEMS

System	*2x1*		*2x2*		*2x4*		*4x2*		*4x4*	
BER	10^{-4}	10^{-5}	10^{-4}	10^{-5}	10^{-4}	10^{-5}	10^{-4}	10^{-5}	10^{-4}	10^{-5}
No CSIT	24dB	27dB	14dB	17dB	8dB	10dB	11dB	13dB	7dB	8dB
Full CSIT	17dB	22dB	7dB	9dB	4.5dB	6.5dB	4dB	6dB	0.5dB	1.5dB
Mean CSIT $\sigma_e^2 = 0.01$	22dB	26dB	13dB	15dB	7dB	9dB	5dB	7dB	3.2dB	4.5dB
Mean CSIT $\sigma_e^2 = 0.1$	26dB	>30dB	13.5dB	16dB	7.5dB	9.5dB	6dB	8dB	3.7dB	5dB
Covariance CSIT	29dB	35dB	14dB	18dB	-	-	-	-	-	-

that can be obtained by optimal antenna selection, beamforming and power loading.

In the next simulation we analyze the error performance of two transmissions that have the same spectral efficiency and the same diversity order. To do this an appropriate modulation scheme has to be chosen for each space-time codeword matrix. The spectral efficiency that is simulated is $\eta = 2bits / s / Hz$. For this, a rate one space-time code is combined with 4-PSK modulation and a rate ½ code employs a 16-QAM modulation. In TABLE 2 are the results obtained during simulations.

TABLE 2: PERFORMANCE TRANSMISSION WITH EQUAL SPECTRAL EFFICIENCY

System and modulation	*2x4 4-PSK*		*4x2 16-QAM*	
CSIT	*no CSIT*	*mean CSIT*	*no CSIT*	*mean CSIT*
$BER = 10^{-4}$	8dB	7dB	19dB	9dB
$BER = 10^{-5}$	10dB	9dB	21dB	10dB

For applications that require a fix spectral efficiency the transmission with 2 transmit antennas outperforms the transmission with 4 antennas, especially if there is no CSIT, the 2x4 systems is with 10dB better that the other. These results are due to the fact that QPSK modulation is more robust than 16-QAM against the influence of noise and the rate of the *2x4* system is twice the rate of the *4x2* system. When CSIT is available at the transmission, due to precoding gain and higher transmit diversity order, the performances of the two systems are almost equal, the system with 2 transmit antenna outperforms the system with 4 transmit antennas only with 1dB.

V. CONCLUSIONS

By exploiting the channel knowledge at the transmitter, the channel capacity and the system error performance are significantly improved even when the channel is correlated. Statistical channel information (mean or covariance) allows the precoding of the data, so that the transmission can be adapted to the channel conditions.

If CSIT is available a higher number of transmit antennas ensures better error performances.

For fix spectral efficiencies, the higher gain compared to the transmission with no CSIT are obtained for higher diversity orders even if the code rate is smaller.

Choosing the right form of CSIT depends on the MIMO channel characteristics and the feedback channel rate and delay. If the channel is slow time varying the precoding based on the mean channel matrix estimation can be implemented, but if the channel is fast time varying, covariance based CSIT is more appropriate.

It must be stated that all the transmissions with CSIT outperforms the STBC transmissions with no channel adaptability.

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